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INVERSION OF THE LUNAR TRAVEL TIME DATA

by

© EDWARD JOHN ROEBROEK

A THESIS

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IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled INVERSION OF THE LUNAR TRAVEL TIME DATA submitted by Edward John Roebroek in partial fulfilment of the requirements for the degree of Master of Science in Geophysics.

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Supervisor

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Date : March 4 , 1974

ABSTRACT

An investigation of the body wave travel time data obtained from the Apollo program was undertaken. The main effort was directed at finding all velocity depth profiles consistent with the travel time data. This P and S wave data sampled the first 100 km of the lunar interior. Using a radially symmetric velocity structure as an approximation to the moon this lunar seismic data was inverted by the Hedgehog method. This method was used for the investigation because of its ability to deal with both nonlinear inversion and uncertainties in the data.

Differing assumptions on the accuracy of the lunar data and on some interpretations of body wave arrivals were made to see what effects these had on the acceptable range of velocity structure the Hedgehog algorithm produced. These were then critically examined. The best P wave velocity structure obtained was one containing the following:

1) A step-wise velocity increase in the first 4 km to a velocity of 4 km/sec.
2) A continuous increase to a velocity of 4.5 km/sec at 14 km depth.
3) A rise in velocity to 6.5 km/sec at 20 km depth.

4) A constant velocity layer of 6.5 km/sec extending to 60 km depth.

5) A rise to a high velocity lid of 8.5 km/sec at a depth of 60 km.

6) A drop to a constant velocity half space of 7.75 km/sec below 80 km depth.

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Figures 3a, 3b, 3c are from C.H. Chapman (1973). Figure 6 and the data in Tables 1, 2 and 3 were obtained through the courtesy of A. Dainty.

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CHAPTER 1

INTRODUCTION

The basic problem of geophysics may be stated in the following manner: Given j observations E_i^0 on the earth (gross earth data) and given an earthmodel $\vec{m}$ of n physically interesting parameters that describe a set of properties of the earth, and given a set of rules that assign to an earthmodel a set of theoretical observations what can we determine about our earthmodel $\vec{m}$? Or rephrasing this to: Given an incomplete set of observations on the earth what can we say about its structure?

This is the statement of the inverse problem of geophysics which this thesis will examine, especially with regard to the practical aspects of the inversion of seismic data to obtain an earthmodel. Seismic data is often inaccurate, inconsistent, and occasionally incorrect. I will examine some direct inversion techniques and some inversion methods that rely on a search through an earthmodel parameter space.

The direct inversion techniques to be examined shall include:

1) Herglotz-Wiechert type inversion which involves the inversion of an exactly known travel time $(T-\Delta)$ curve to obtain a seismic velocity profile of the earth.

2) Linearization procedure due to Backus and Gilbert (1967-1968 a,b) which uses a linear perturbation theory to obtain a best fit model and then examines the trade off between variance and resolution in the model. Jackson (1972) has examined the non-uniqueness of the solutions for the linearization technique by examining the marginally acceptable models - the Edgehog technique.

The direct techniques to be examined involve parameterization of a set of earthmodels. A parameter vector (a point in parameter space) thus defines a particular earthmodel. The task is then to find all acceptable points in parameter space with an acceptable point being one which produces theoretical observations that agree with the actual observations. Two methods are in common use:

1) Monte Carlo search which is a random search in parameter space.

2) Hedgehog which is a controlled search from a starting point and delineates an entire connected region of acceptable points.

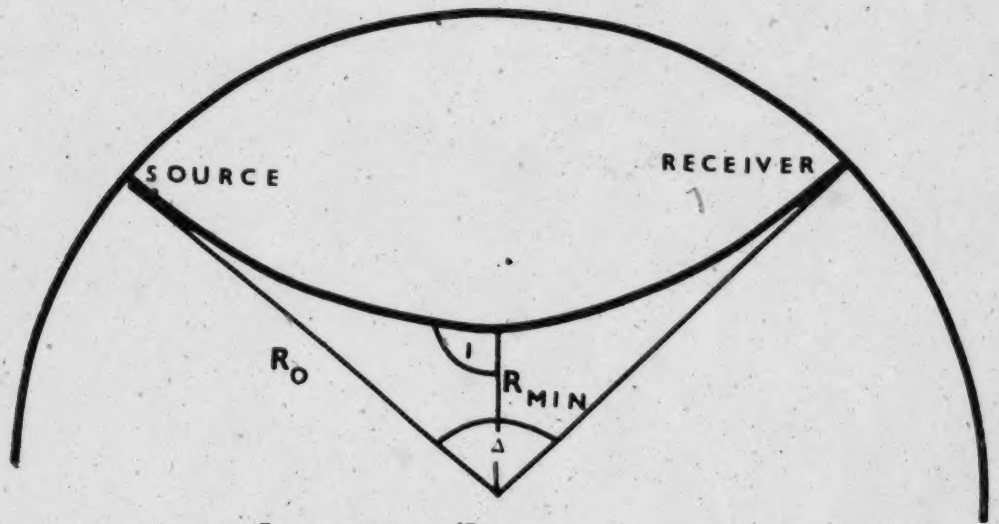
It should be noted that in seismology the observables are the travel time T (the time it takes seismic energy to propagate from a source to a receiver), the ray parameter $p = dT/d\Delta$, where Δ is the earth centre angle (Figure 1), and the amplitude of the seismic wave at the receiver $A \sim d^2T/d\Delta^2$. Due to the generally high

frequency of seismic waves with respect to velocity structure (velocity as a function of depth) in the earth the travel time may be described in terms of the theory of geometrical optics with exceptions where diffraction, interference, or tunneling become important.

Applying geometrical optics we obtain the set of relations in Figure 1 for a seismic wave whose source is on the earth's surface in a radially symmetric and isotropic earth.

The theories of evolution and genesis of the moon are modernized equivalents of theories that fall into three broad categories. That is fission from the earth, capture, or formation of the moon from the solar nebula in an orbit that is in close proximity to the earth.

The only restriction that seismic data puts on these theories is that at least some portions of the moon were molten and differentiation resulted.



$$p = \frac{r}{v(r)} \sin i(r) = \frac{dT}{d\Delta}$$

$$\Delta(p) = 2 \int_{r_{min}}^{r_0} \frac{p\, dr}{\sqrt{r^2/v^2(r) - p^2}}$$

$$T(p) = 2 \int_{r_{min}}^{r_0} \frac{r^2/v^2(r)\, dr}{\sqrt{r^2/v^2(r) - p^2}}$$

where

p	is the ray parameter; a constant along a particular ray;
r	is radial position;
$v(r)$	is the velocity structure;
$\Delta(p)$	the earth centre angle between source and receiver;
r_0	the radius of the earth;
r_{min}	the turning point of the ray.

Figure 1

Seismic Ray Theory

CHAPTER 2

DIRECT INVERSION

The traditional method of inversion for a travel time curve is due to Herglotz (1907) and Wiechert (1910) with the solution (Jeffries 1962) given by:

$$(1) \qquad \pi \ln\left(\frac{r_{max}}{r_{min}}\right) = \int_0^{\Delta(p_1)} \cosh^{-1}\left(\frac{p}{p_1}\right) d\Delta$$

with p_1 the value of the ray parameter for the ray bottoming out at $r_{min}(p_1)$. (See Figure 1 in the introduction for notation.) This is an exact solution. If you know the structure above r_{min} you can find the structure at r_{min}. Thus one works down from the surface.

This method has an inherent limitation in that the ray parameter must be known for all values of Δ between 0 and $\Delta(p_1)$ and this method fails in the presence of a low velocity zone (Figure 2).

The cases where there are low velocity zones may be taken care of by an extension of this method due to Gerver and Markushevich (1966, 1967, 1968). It is best described in Chapman (1973) and is summarized below.

We know that $\Delta(p)$ can be written as below

$$(2) \qquad \Delta(p) = \int_{r_{min}}^{r_0} \frac{dr}{r(\eta^2 - p^2)^{\frac{1}{2}}} \qquad \text{where} \quad \eta = r/v$$

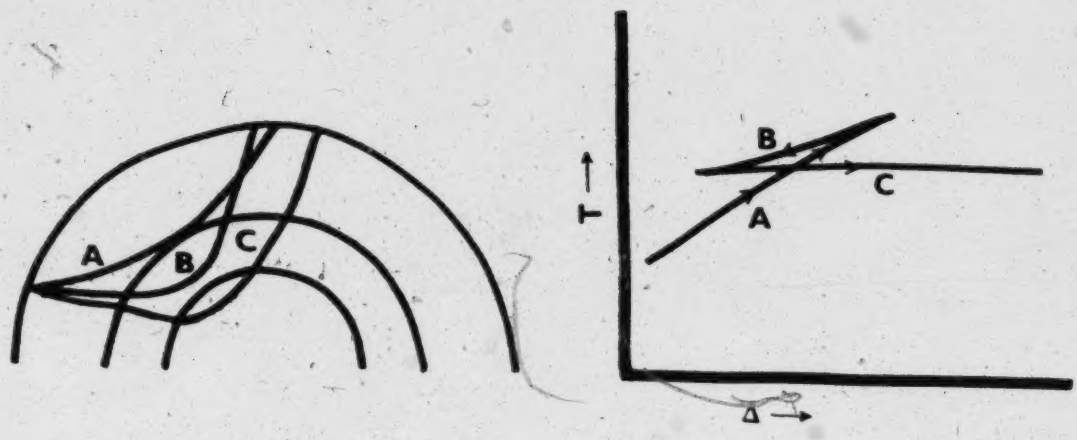


Figure 2a)

Triplication

A triplication occurs when there is a zone of high velocity gradient ($dv/dr > v/r$) which is characterized on the travel time curve by three branches. Often these separate branches are so close in time as to be indistinguishable.

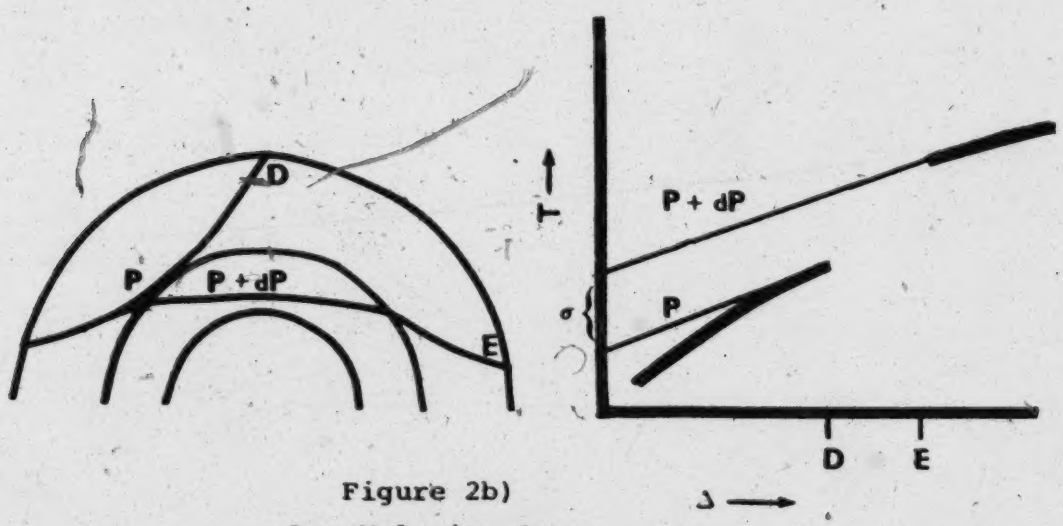


Figure 2b)

Low Velocity Zone

When there exists a layer in the earth where $dv/dr > -v/r$ this is called a low velocity zone (LVZ). Two rays with ray parameters P and p + dP as in the figure will surface with a large separation. Hence a shadow zone (D-E) in the travel time curve where no observations ($T = \infty$) exist.

applying the integral operator

$$(3) \qquad \int_{p_I}^{\eta_0} \frac{dp}{(p^2 - p_I^2)^{\frac{1}{2}}} \qquad \text{where} \quad \eta_0 = r_0/v_0$$

to $\Delta(p)$ we obtain

$$(4) \qquad \int_{p_I}^{\eta_0} 2p \left(\int_{r_{min}}^{r_0} \frac{dr}{r(\eta^2 - p^2)^{\frac{1}{2}}} \right) \frac{dp}{(p^2 - p_I)^{\frac{1}{2}}} \quad .$$

This is an integral in the p-r plane where this integral is over the area D_q^0, with the area between $\underline{r_k}$ and $\overline{r_k}$ a low velocity zone. For simplicity only one low velocity zone was shown in Figure 3a. Let

$$(5) \qquad D(q) = D^0(p_I) + \sum_{k=1}^{k^*} D^k$$

with $p_{k+1}^* \leq p_I < p_k^*$ and k^* being the total number of low velocity zones. Reversing the order of integration we obtain Figure 3b.

$$(6) \qquad D(q) = \int_{r_{min}(p_1)}^{r_0} \frac{2}{r} \left(\int_{p_I}^{\eta(r)} \frac{p\, dp}{(\eta^2 - p^2)^{\frac{1}{2}}(p^2 - p_I^2)^{\frac{1}{2}}} \right) dr \quad .$$

Evaluating this integral we get

$$(7) \qquad D(q) = \int_{r_{min}}^{r_0} \frac{\pi}{r}\, dr = \pi \ln\left(\frac{r_0}{r_{min}}\right) \quad .$$

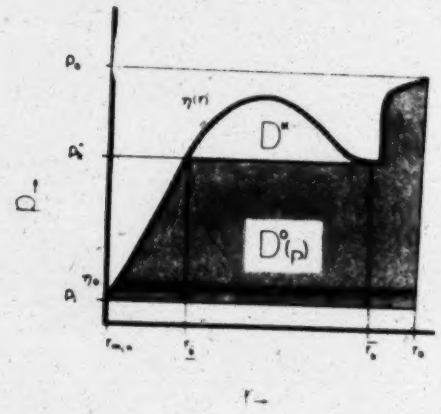


FIGURE 3a

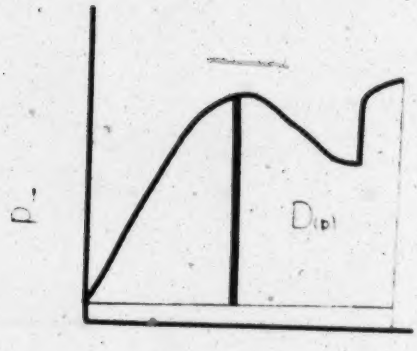


FIGURE 3b

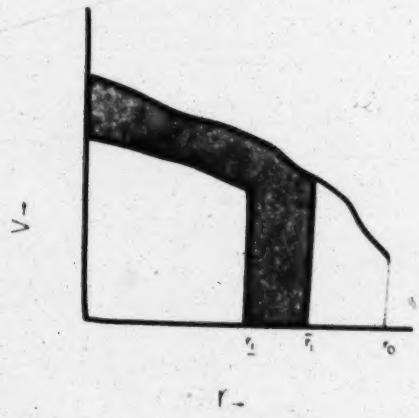


FIGURE 3c

Setting it equal to the first integral we get

$$(8)\qquad \pi \ln\left(\frac{r_0}{r_{min}}\right) = \iint_{D(q)} \frac{2p\ ds}{r(\eta^2 - p^2)^{\frac{1}{2}}(p^2 - p_I^2)^{\frac{1}{2}}}$$

$$= \iint_{D_q^0} \frac{2p\ ds}{r(\eta^2 - p^2)^{\frac{1}{2}}(p^2 - p_I^2)^{\frac{1}{2}}}$$

$$+ \sum_{k=1}^{k^*} \iint_{D^k} \frac{2p\ ds}{r(\eta^2 - p^2)^{\frac{1}{2}}(p^2 - p_I^2)^{\frac{1}{2}}}$$

$$= \int_{p_1}^{\eta} \frac{\Delta(p)}{(p^2 - q^2)^{\frac{1}{2}}}\, dp$$

$$+ \sum_{k=1}^{k^*} \int_{\underline{r_k}}^{\overline{r_k}} \frac{2}{r} \int_{p_k}^{\eta} \frac{p\ dp}{(\eta^2 - p^2)^{\frac{1}{2}}(p^2 - p_I^2)^{\frac{1}{2}}}$$

$$= \Delta(p)\ \cosh^{-1}\left(\frac{p}{p_1}\right) - \int_{p_1}^{\eta} \frac{d\Delta}{dp} \cosh^{-1}\left(\frac{p}{p_I}\right) dp$$

$$+ \sum_{k=1}^{k^*} \int_{\underline{r_k}}^{\overline{r_k}} \frac{2}{r} \tan^{-1}\left(\frac{\eta^2 - p_k^2}{p_k^2 - p_I^2}\right)^{\frac{1}{2}} dr$$

but $\Delta(\eta_0) = 0$ and $\cosh^{-1}(1) = 0$.

Thus:

$$(9)\qquad \pi \ln\left(\frac{r_0}{r_{min}}\right) = \int_0^{\Delta(p_I)} \cosh^{-1}\left(\frac{p}{p_I}\right)\, d\Delta$$

$$+ \sum_{k=1}^{k^*} \int_{\underline{r_k}}^{\overline{r_k}} \frac{2}{r} \tan^{-1}\left(\frac{\eta^2 - p_I^2}{p_k^2 - p_I^2}\right)^{\frac{1}{2}} dr$$

where the integration is with respect to all branches of the travel time curve. That is in the case where there are several branches then the integration is positive on advancing branches and negative on receding branches.

Examining the case where k* is zero (there are no low velocity zones) then we see that we have a solution of the form

$$(10) \qquad \pi \ln\left(\frac{r_0}{r_{min}}\right) = \int_0^{\Delta(p_I)} \cosh^{-1}\left(\frac{p}{p_I}\right) d\Delta \quad .$$

Knowing an exact and complete T-Δ curve we can obtain $p(\Delta) = dT/d\Delta$. If we know the structure above any p_I we can integrate (1) and solve for r_{min}. Since $v(r_{min}) = p_I/r_{min}$ we then know the structure at this point, and using the same procedure we can find the structure for $p_I + dp_I$.

For the case where k* = 1 we have one low velocity zone and we must evaluate the term

$$(11) \qquad \int_{\underline{r_1}}^{\overline{r_1}} \frac{2}{r} \tan^{-1}\left(\frac{\eta^2 - p_1^2}{p_1^2 - p_I^2}\right) dr \quad .$$

Above the low velocity zone the only term to appear is that of equation (10). Thus $\overline{r_1}$ (the upper bound of the low velocity zone) can be determined. Since this is a

low velocity zone we can put a maximum upper bound on the velocity and η. From the travel time curve we can measure the time jump σ_k across the shadow zone due to the low velocity zone. This is a measure of the extra time through the low velocity zone. Knowing this and the maximum velocity a least lower bound can be placed on r_1. Thus our solution will look as in Figure 3c.

Gerver and Markushevich (1967) have shown that through any point in this solution space (the shaded area) there exists one solution which goes through that point. Note that not all curves lying within the shaded region are solutions. For any curve through a low velocity channel the curve to the next channel is uniquely determined.

There are several practical difficulties in the application of this method which make it unsuitable for inversion of travel time data. These are:

1) There may exist branches (triplications) on the travel time curve. This method requires exact knowledge of all ray parameters on all branches in a given range which may be impossible since the different branches of a travel time curve are often inseparable on the seismic record.

2) The ray parameter $p = dT/d\Delta$ is required. Unless we are dealing with arrays where a direct measurement of p can be made we are

forced to differentiate the travel time curve. Numerical differentiation of experimental data is inaccurate. This is especially important where the travel time curve flattens out and where a small uncertainty in p would lead to large uncertainties in a resulting earthmodel.

3) This method is based on classical ray theory which is inadequate in regions of high velocity gradient due to diffraction. This diffraction is frequency dependent leading to dispersion in the wavetrain. Thus for example P and PcP data cannot be simultaneously inverted by this method to obtain the core radius.

4) Uncertainties in the data are not taken into account. We have assumed a complete and accurate knowledge of the travel time curve. Unfortunately seismic data is a nonrandom, inaccurate sampling of a travel time curve. Thus to make use of this method we must fit a curve to our data hoping that it is a near enough approximation to our travel time curve to calculate an accurate earth-model.

LINEAR INVERSE THEORY

The statement of the inverse problem in geophysics is the following: Given J measured gross earth data E_1^0, E_2^0, ... E_J^0 with each E_i^0 a function only of an earthmodel (the data has been corrected for instrumentation effects) $m(r) = (a_1(r), a_2(r), \ldots a_N(r))$ what do we know about each $a_i(r)$? For example the earth's mass, moment of inertia, travel time between two points of P or S waves are all gross earth data. The a_i's are physically interesting functions of the earth, generally but not necessarily assumed to be functions of radius such as density ($\rho(r)$), or P wave velocity ($\alpha(r)$).

Let us define an earthmodel m to be an ordered N-Tuple of functions on $0 \leq r \leq 1$ with

$$\vec{m} = (a_1, a_2, \ldots, a_N) \tag{12}$$

and define another earthmodel $\vec{m}'$

$$\vec{m}' = (a_1', a_2', \ldots, a_N') \tag{13}$$

Then we can define a multiplication by a scalar and addition as

$$b\vec{m} + b'\vec{m}' = (ba_1+b'a_1', ba_2+b'a_2', \ldots, ba_N+b'a_N') \tag{14}$$

We can also define an inner product

$$(\vec{m},\vec{m}') = \int_0^1 dr[a_1(r)a_1'(r)+a_2(r)a_2'(r)+\ldots a_N(r)a_N'(r)] \tag{15}$$

$$\text{and} \quad ||\vec{m}|| = (\vec{m},\vec{m})^{\frac{1}{2}} .$$

With these definitions and the realization that we can restrict ourselves to deal with only a finite set of physically interesting functions that are piecewise continuous and bounded then

$$(16) \qquad \sum_{i=1}^{N} \int_{0}^{1} ||a_i(\vec{r})||d\vec{r} < \infty$$

which means that we can complete this space to form a Hilbert space M_e.

It should be noted that in equation (15) N positive weighting functions, depending on $\vec{r}$, may be inserted. For example if a model is comprised of a series of concentric shells we may wish to weight the shells with a value dependent on their width.

It should be obvious that the ith gross earth datum is a rule which assigns to every $\vec{m} \in M_e$ a real number $E_i(\vec{m})$. In general then a gross earth datum will be a nonlinear functional on M_e. Since linear functionals are easier to deal with than nonlinear functionals consider the example of the following linear functionals. The mass E_1 and the moment of inertia E_2 of the earth

$$(17) \qquad E_1 = 4\pi \int_0^1 dr\ \rho(r) \quad ; \quad E_2 = \frac{8\pi}{3} \int_0^1 dr\ r^2\rho(r) \ .$$

If a measurement of a particular gross earth datum E_i for the real earth is E_i^0 then we know that the point $m_e \in M_e$ which describes the real earth lies on the hypersurface defined by

$$E_i(\vec{m}) = E_i^0 \quad . \tag{18}$$

If $E_i(\vec{m})$ is a linear function then the solution space will be a hyperplane.

If we have $E_1^0, E_2^0, \ldots, E_j^0$ gross earth data then we know that the real earth lies on the intersection in M of the J hypersurfaces.

$$E_j(\vec{m}) = E_j^0 \tag{19}$$

Thus it can be seen that solely from the measurements E_i^0 we can deduce nothing about the earth. To be able to make deductions we need to know the j functionals $E_j(\vec{m})$. The importance of knowing the $E_i(\vec{m})$ cannot be overstressed (Gerver 1973) since any errors or approximations inherent in them will be carried throughout the problem.

The Problem of Non-Uniqueness

Since M is an infinite dimensional space and equation (19) introduces only a finite number of restrictions intuitively we can see that our solution set

will be an infinite dimensional subset of M. Consider the following pathological example.

$$(20) \quad E(\vec{m}) = n + \int_0^1 dr[(a_1-1)^2 + (a_2-2)^2 + \ldots (a_n-n)^2]$$

Then the observation $E^0(\vec{m}) = n$ imposes only one condition yet the solution is the point

$$(21) \quad a_1 = 1, \quad a_2 = 2, \ldots, a_n = n \quad .$$

To remove such mathematical pathologies as this, we define Frechet differentiability. E is Frechet differentiable at $\vec{m}$ if for any $\delta\vec{m}$ we have

$$(22) \quad E(\vec{m}+\delta\vec{m}) = E(\vec{m}) + (M,\delta\vec{m}) + \varepsilon(\delta\vec{m})$$

where

$$\lim_{\delta\vec{m}\to 0} \left(\frac{\varepsilon(\delta\vec{m})}{||\delta\vec{m}||}\right) = 0$$

and $\vec{M} \in M_e$

$$M_j = \partial E/\partial a_j \quad .$$

$\vec{M}$ is called the Frechet kernel of E at $\vec{m}$, and it should be noted that equation (22) is the first terms of a Taylor series expansion. The condition that small perturbations m cause small changes in observable E is often violated in geophysics. The travel time curve with a low velocity zone is such an example since small

perturbations in depth to low velocity zone causes a small shift in location of the shadow zone which implies that certain points on the travel time curve go from a finite to an infinite value no matter how small the perturbation.

Let $\vec{m}^{(1)}$ be a starting model that we guess and let $\vec{m}^{(1)} + \delta\vec{m}^{(1)}$ be the true earthmodel in that it furnishes all the observed data within experimental accuracy. Then we seek to minimize

$$(23) \quad ||\vec{m}^{(1)}-\vec{m}^{(1)}-\delta\vec{m}^{(1)}|| = \int_0^1 dr[(a_1'-a_1)^2 + (a_2'-a_2)^2 + \ldots\ldots (a_n'-a_n)^2]$$

where $\vec{m}^{(1)} + \delta\vec{m}^{(1)} = (a_1', a_2', \ldots, a_n')$

$$= (a_1+\delta a_1, a_2+\delta a_2+\ldots, a_n+\delta a_n)$$

and $\vec{m}^{(1)} = (a_1, a_2, \ldots, a_n)$.

To 1st order we can write from equation (22)

$$(24) \quad E_j(\vec{m}^{(1)}+\delta\vec{m}^{(1)}) = E_j(\vec{m}^{(1)}) + (\vec{M}_j^{(1)}, \delta\vec{m}^{(1)}) .$$

If $\vec{m}^{(1)}$ and $\vec{m}^{(1)}+\delta\vec{m}^{(1)}$ are so close that we can ignore ϵ in (22) then by the variational technique

$$(\delta\vec{m}) = \sum_{i=1}^{N} \alpha_i^{(1)} \vec{M}_i^{(1)}$$

with $\alpha_i^{(1)}$'s given by

$$\sum_{k=1}^{n} (\vec{M}_j^{(1)}, \vec{M}_k^{(1)})\alpha_k^{(1)} = E_j(\vec{m}^{(1)}+\delta\vec{m}) - E_j(\vec{m}^{(1)}) - (M_j^{(1)}, \delta\vec{m}^{(1)}) .$$

Neglecting ε in (22) implies that $\delta\vec{m}^{(1)}$ may not be exact, but the above process can be turned into an iterative process with an $\vec{m}^{(n+1)}$ defined as

$$(25) \quad \vec{m}^{(n+1)} = \vec{m}^{(n)} + \sum_{k=1}^{j} \alpha_k^{(n)} M_k^{(n)} .$$

If (24) were exact then

$$(26) \quad E_j(\vec{m}^{(2)}) = E_j^0$$

and $\vec{m}^{(2)}$ would be our exact solution but as it is all we can say in our iterative process is that

$$(27) \quad |E_j(\vec{m}^{(n+1)}) - E_j^0| < |E_j(\vec{m}^{(n)}) - E_j^0|$$

and $$|\delta\vec{m}^{(n+1)}| < |\delta\vec{m}^{(n)}| .$$

Thus the above technique gives an iterative method for finding a solucion.

Let us restrict ourselves to a set of models w where in equation (24) ε is small enough to be neglected. Assume that $\vec{m}_0$ the true earth lies in the set of models. Then the natural way to assess the adequacy of a model $\vec{m}$

is in terms of the resolving power. If we assume that at some r_0 the data provides an estimate of the true gross earth data that is a smoothed average of the structure around r_0 then for a linear functional a plausible and attractive smoothing is of the form

$$(28) \quad \langle \vec{m}(r_0) \rangle = (\vec{A}(r_0), \vec{m}_0) = \int_0^1 \vec{A}(r, r_0) \cdot \vec{m}_0(r) dr$$

and $\langle \vec{m}(r_0) \rangle$ is an estimate of the true earth $\vec{m}_0(r_0)$. It can be seen that $\vec{m}(r_0) = \vec{m}_0(r_0)$ if $\vec{A}(r_0) = \delta(r-r_0)$. Thus $\vec{m}(r_0)$ is a good estimate of the true earth if $A(r,r_0)$ as a function of r has a peak at r_0. The sharper the peak - nearer to a delta function - the better the estimate.

Under the criterion of linearization Backus and Gilbert (1968) have shown that

$$(29) \quad \vec{A}(r_0) = \sum_{i=1}^{N} b_i(r_0) \vec{M}_i$$

with the b_i chosen such that A is unimodular.

$$(30) \quad \int_0^1 dr \, \vec{A}(r, r_0) = 1 \quad .$$

We will define a measure of the width of A by a quantity $S(r_0, A)$ called the spread of A by:

$$(31) \quad S(r_0, A) = 12 \int_0^1 (r-r_0)^2 A^2 dr \quad .$$

It is obvious that an A that makes a small spread $S(r_0,A)$ is a good resolution function. This resolution function and our estimate of it (29) solve the problem of non-uniqueness since all models within the linear regime satisfying the data give the same estimate $\langle\vec{m}(r_0)\rangle$ for $\vec{m}_0(r_0)$.

Associated with each estimate $\langle\vec{m}(r_0)\rangle$ there is introduced a standard error calculated from the observational errors. It can be shown (Backus and Gilbert, 1968) that the smaller the spread S the larger the standard error at any radius r_0. This is a trade off between precision and resolution. Thus there does not exist a best set of $b_i(r_0)$ but instead a relationship between the spread and error. This is the familiar power-spectrum trade off where you playoff the resolution and the precision. Whatever compromise is used in a particular case depends on the desired results. The relationship between accuracy and resolution is a function of depth and model as well as the error estimates as is shown below. The spread S can be formed from a quadratic as below:

$$(32) \qquad S = \sum_i \sum_j b_i b_j S_{ij}$$

where S_{ij} is a coefficient matrix from the inner product of the Frechet kernels, i.e.

(33) $S_{ij} = (\vec{M}_i, \vec{M}_j)$.

The variance $(\Delta m)^2$ of the average $\langle \vec{m}(r_0) \rangle$ is given by

(34) $(\Delta m)^2 = \sum_i \sum_j b_i b_j C_{ij}$

where C_{ij} is the covariance matrix between E_i^0 and E_j^0. It can be seen that for statistically independent data C_{ij} is a diagonal matrix. We construct a third quadratic below with w being a constant

(35) $B_{ij} = S_{ij}\cos\theta + C_{ij} w \sin\theta$.

For every θ we can find a b_j which minimizes B_{ij}. We note the following. When θ is small B_{ij} resembles S_{ij} and we will generate delta like functions that have a minimal S but will have an unacceptably large $(\Delta m)^2$. As θ increases to $\pi/2$ S will increase and $(\Delta m)^2$ will decrease.

The Edgehog

The variance is a statistical way of speaking of the extent of our model set about our "best fit" model. Upon examination, the variance about our best fit model does not really give us the boundaries to our model space. As was seen previously the solution is the space

within N intersecting hypersurfaces and thus has definite boundaries to within experimental accuracy. Let us now then try to find the boundaries to our solution space. That is the marginally acceptable solutions (Jackson 1973).

Let us assume that we have observed n gross earth data E_i^0 and that we have derived j good enough relations between a model $\vec{m} = (m_1 \ldots m_m)$ and the observations such that

$$E_i = E_i(\vec{m}) \quad .$$

Then we can define an error ε_i as

$$\varepsilon_i = E_i^0 - E_i(\vec{m}) \tag{36}$$

which can be expanded in a 1st order Taylor series as below

$$\varepsilon_i \simeq E_i^0 - E_i(\vec{m}^{(1)}) - \sum_{j=1}^{m} \left(\frac{\partial E_i}{\partial m_j^{(1)}}\right)(\delta\vec{m}^{(1)}) \tag{37a}$$

where we have as in equation (23) $\vec{m} + \delta\vec{m}^{(1)} = \vec{m}^{(1)}$. That is we have a true earth $\vec{m}$ which we by a priori knowledge have modeled as $\vec{m}^{(1)}$ and are incorrect by $\delta\vec{m}$. This is equation (23) rewritten with an error term. Which when rewritten in matrix form is

$$\underset{\sim}{\varepsilon} = \underset{\sim}{y} - \underset{\sim}{A}\,\underset{\sim}{x} \tag{37b}$$

where $\underset{\sim}{\varepsilon}$ is the vector of the ε_i's

$\underset{\sim}{y}$ is the vector of the $y_i = E_i^0 - E_i(\vec{m})$

$\underset{\sim}{A}$ is the matrix of the $A_{ij} = \partial E_i / \partial m_j = \partial E_i / \partial m_j$

$\underset{\sim}{x}$ is the vector of the δm_j .

If we now work with standardized data - data with zero mean and a unit variance - then as was previously shown our model space - the set of models that will produce the observed gross earth datum - is non-uniquely described by an acceptable model with a variance. Thus our solution space consists of the models with an r.m.s. residual r given by

$$(38) \qquad r \equiv \frac{1}{\sqrt{n}} \sqrt{\sum_{i=1} \varepsilon_i^2} \leq 1 \quad .$$

In choosing an acceptable model we minimize r for a given resolution. We have also applied an averaging process to our model by some averaging distribution as in equation (28) which states that as long as we have a $\delta\vec{m}$ small enough - i.e. a model not too far away from a true earth - the averaging over the model will give us gross earth data that are within the acceptable bounds of the observed data. For standardized data this can be written as:

$$(39) \qquad s = \sqrt{\frac{1}{m} \sum_{j=1}^{m} m_j^2} \leq 1 \quad .$$

Equations (38) and (39) then correspond to conditions on the variance and the resolution.

In the Edgehog procedure we seek marginally acceptable solutions. That is we wish to know the bounds of our solution space. Thus we must solve the equation

$$r = 1 \quad \text{and} \quad s \leq 1$$

or

$$r \leq 1 \quad \text{and} \quad s = 1 \ .$$

This linear inverse theory, as applied to seismology, though very attractive, has one great practical limitation. That is the assumption that small perturbations in model $\delta\vec{m}$ cause small changes in observable E_i. Such is the case where there are no low velocity zones. Even where there are no low velocity zones we have the added problem of numerical precision in our calculations. That is small enough perturbations $\delta\vec{m}$ used in a numerical calculation may produce an $E_i + \delta E_i$ with a numerical uncertainty $> \delta E_i$.

Linear inverse theory is attractive in that:

1) Incomplete data may be used. Points on a travel time curve rather than the curve itself may be inverted.

2) Inaccuracies in the data are taken account of and appear as uncertainties in the models produced.

3) The "rules" used for the calculation of the observables from the earth model are not restricted to classical ray theory but can be any physically attractive set. Thus if we find we are dealing with a nonlinear set of observables such as a travel time curve we can apply a transformation to obtain a linear set of observables. For example, we can examine the τ-p rather than the T-Δ space of observations (Bessonova et al 1974).

CHAPTER 3

INDIRECT INVERSION

Basically the indirect method of inversion involves a search through a parameter space; each point in this parameter space being an earthmodel. These parameters $p_1, p_2, \ldots, p_M$ are related to the earthmodel $\vec{m} = (a_1(\vec{r}), a_2(\vec{r}), \ldots, a_N(\vec{r}))$ of the previous chapter in the following way. The $a_i(\vec{r})$ can be approximated by a set of coefficients and a function. That is

$$a_i(\vec{r}) = f_i(p_j \ldots p_L, \vec{r})$$

where the p_i's are not functions of $\vec{r}$. Knowing the parameters and the functions we then have an earthmodel. Thus the parameters themselves describe an earth, and any point in the parameter space describes an earthmodel. For example if we are interested in the inversion of travel times we may choose a N layer earth with 2N parameters; the parameters being the depth to the ith layer and velocity in the ith layer. Any set of physically interesting and numerically compact quantities may make up a parameter space and thus a set of earthmodels.

The search in parameter space is to find the set of all acceptable points in this space. An

acceptable point being one which produces a set of theoretical observables that matches within uncertainties a set of physical observations. This set of acceptable points in parameter space is our solution.

The actual detailed process of the finding of acceptable parameter space points is diagrammed in Figure 4. The following is condensed from (Keilis-Borok and Yanovskaya, 1967).

Observational Data

This is the input of the physical pertinent data to a particular problem in geophysics. We wish to find all earthmodels that will, to within experimental uncertainty, produce theoretical observations to match these observations.

A priori limits

Fortunately we do not begin our search of parameter space without any a-priori data. That is we know from previous physical observation, or deduction, or by the physical laws themselves, that we have some bounds on our parameter space wherein lie all acceptable models. We know in what broad limits we can find the seismic velocity structure of

SCHEMATIC OF INDIRECT INVERSION

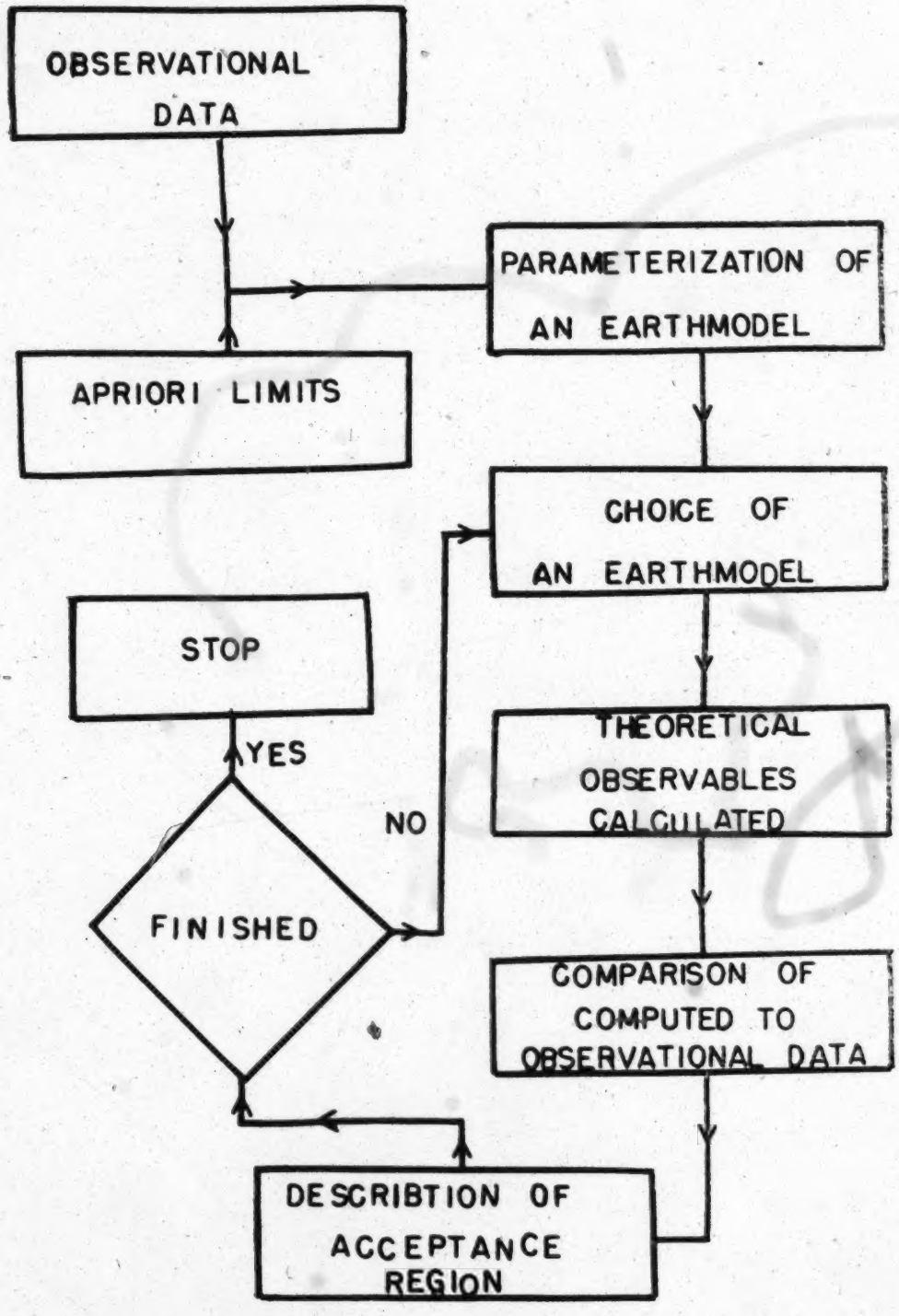


FIGURE 4

the earth. Thus we need not search all of parameter space but only a section thereof. This imposition of a-priori limits is very important. If we choose broad limits then our calculations of earthmodels and observables may be too costly. If we choose a-priori limits that are too narrow we will have excluded acceptable points in parameter space from our search and may miss physically significant earthmodels.

Parameterization of Earthmodels

We can approximate a true earth by a finite set of parameters. The choice of parameters depends upon the problem and data being considered. For example in the case of inverting seismic data (P or S wave travel time, or ray parameter, or amplitude, or surface wave dispersion, or free oscillations etc.) the parameters usually chosen are some set of descriptors of $\alpha_n(r)$ (P wave velocity in the nth layer), $\beta_n(r)$ (S wave velocity in the nth layer), $\rho_n(r)$ (density in the nth layer), $Q_n(r)$ (attenuation in the nth layer), L_n (location of the nth layer). Thus the parameters will be physical relevant descriptors of some set of functions which describe the earth. We must avoid two extremes. A too simple parameterization wherein we would miss physically relevant

models and a too complicated parameterization which would be too costly in terms of computer time.

Choice of an Earthmodel

Recall that we are looking for a region, or set of regions, in parameter space that contain only acceptable points. The only practical way to accomplish this is to use an N dimensional net (with N the dimension of the parameter space) and instead of searching an infinity of points in a region just to examine the points on the nodes of the net.

The step size of this net in any dimension is obviously important. Too fine a step size and the computations become too costly; too coarse a step size and we may lose information.

It would be desirable to examine all nodes on this net. Generally this is inefficient. Instead we may proceed in one of two ways:

Random Search - Monte-Carlo Inversion

This involves the picking of random points in parameter space and has the advantage (Press 1968) that very little subjective bias in the choice of models is introduced. A good random sampling of parameter space bounded by a-priori limits is

searched, hopefully without the imposition of any subjective physical bias. The disadvantages of this method are that it is extremely costly, and that a random search by its very nature may have missed physically interesting models.

Controlled Search - Hedgehog Inversion

This involves the a-priori knowledge of at least one good node in parameter space (this node might be found by Monte-Carlo search) and then examining in turn all the nth order nearest neighbours of this point. All the neighbours that are accepted are then examined in the same manner until the region of acceptance is completely delineated (Figure 5).

As an example of the two dimensional parameter space Hedgehog search and some of the pitfalls in choosing a wrong grid size examine Figure 5. The heavy lines form a coarse grid, while the light lines form a fine grid and A, B are the regions of acceptance that we wish to find and delineate.

We define the nth order nearest neighbours of a point $\vec{m}$ to be those points $\vec{m} + \varepsilon_i \delta\vec{m}_i$ where $\delta\vec{m}_i$ is the step size in the ith direction, ε_i being ± 1 or 0 and only n, or less, of the $|\varepsilon_i| = 1$.

For example if we found the point 1 (Figure 5) by some method as a good point then on a coarse grid

A HEDGEHOG EXAMPLE

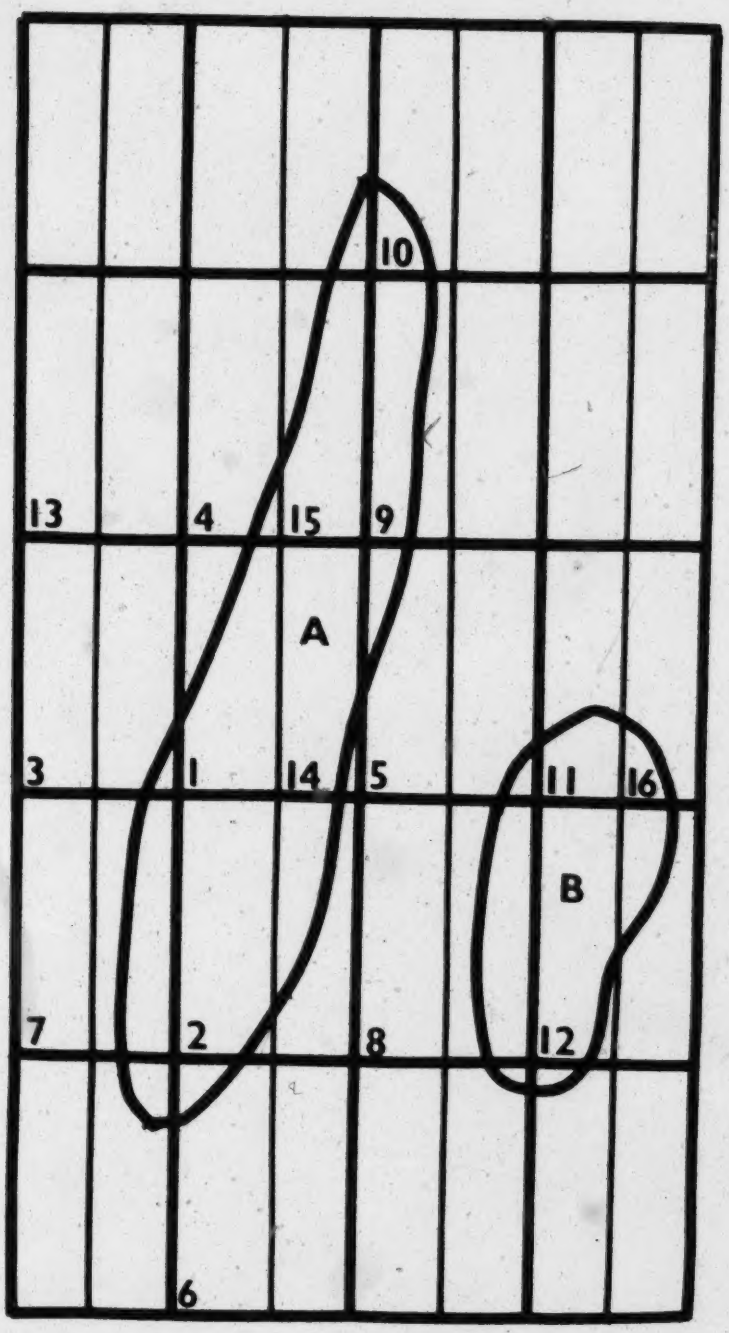


FIGURE 5

would next be examined. The point 2 is an acceptable point. Thus the points 6, 7, 8 would be examined and no more acceptable points would be found. Thus using the coarse grid with an order one search the region of acceptance that would be found would be the points 1 and 2. Region B and the points 9, 10 would have to be found independently.

Using the coarse grid with an order 2 search starting at the point 1 the points 2, 3, 4, 5, 7, 8, 9, 13 would be examined. The good points 2 and 9 would be found. From 2 no good points would be found but from 9 the points 10 and 11 would be found. The region of acceptance that would be found would then include the points 1, 2, 9, 10, 11, 12. We would consider this to be a singly connected region which is false.

Using the fine grid with either an order 1 or 2 search would produce the acceptable points 1, 2, 9, 10, 14, 15.

This illustrates some of the problems inherent in a Hedgehog search. To summarize they are:

1) Only connected or nearly connected regions can be found from a single starting model.
2) Too coarse a grid or too low a neighbour search and you may miss parts of the region.

3) Too find a grid or too high a nearest neighbour search and the cost becomes prohibitive.

Calculation of Theoretical Observations

This is the direct problem of geophysics. That is given an earthmodel, a node in parameter space, calculate the theoretical observables. The calculation and the theory can be whatever is physically attractive.

Comparison of Theoretical to Observational Data

We wish to know when the theoretical observations match our geophysical observations. This is a mathematically inadequately developed problem (V.I. Keilis-Bovok 1970) and is unfortunately highly subjective.

Briefly we wish to know when a set of j observations O_i match a set of j calculations C_i. We will say that we have an acceptable earthmodel when some or all, depending upon the situation, of the following criteria are met:

(1) $$\sqrt{\sum_{i=1}^{j} (O_i - C_i)^2} \leq n_1$$

(2) Only n_2 or less of $|O_i - C_i| \geq n_3$

$$(3) \qquad \left| \sum_{i=1} (O_i - C_i) \right| \leq n_4$$

$$(4) \qquad \sqrt{\sum_{i \in k} (O_i - C_i)^2} \leq n_5$$

where the n_i's are a set of thresholds and k is a subset of observations (i.e. those delineating a triplication).

We are not restricted to this set of acceptance criteria, but can rather choose any physically reasonable set. Thus we need not use a root mean square measure as in (1) but could have gone to an absolute value measure.

When the acceptability of a point in parameter space has been determined, acceptable points are translated to an earth structure and the Hedgehog search continues.

The advantages of using the indirect method are that we are not hampered by our ability or inability to solve a mathematically difficult inverse problem with their attendant assumptions. We are only limited by our knowledge of the physics of the direct problem, which is usually adequate, and by the number of points in parameter space we can afford to search.

A disadvantage is that we must be careful in our parameterization so as not to introduce a subjective bias. As an example of this consider the case where a triplication in a travel time curve is observed. If we have parameterized an earth in such a way that no regions of high velocity gradient (dv/dr) can exist then we will not produce a theoretical triplication. The problem can be more subtle in the case of the existence or non-existence of low velocity zones. Furthermore we do not know if our physical sciences background has limited us so that we may subjectively miss adequate models that do not in some manner meet with our preconceived expectations. Besides this there is the problem that indirect methods of inversion are generally costly. That is they require large amounts of computer time and larger amounts of time in the development of the computer programs.

CHAPTER 4

INVERSION OF THE LUNAR SEISMIC DATA

The Lunar Data

The lunar seismic data has been obtained through the passive seismic experiment (PSE), a set of seismometers deployed by the Apollo missions. Seismic energy was supplied by the impact of the Saturn IVB stage and lunar excursion module for each of these missions on the moon. A full description of the PSE can be found in Latham et al (1972).

Separate seismic phases have been identified on the seismograms (Toksoz et al 1972). If we assume that the moon is radially homogeneous, that is seismic velocity is only a function of radius, then we can invert the travel time curves to obtain the velocity structure of the upper part of the moon. The travel time data has been summarized in Tables 1 and 2 and graphed in Figure 6 (Toksoz et al 1972). The data has been supplied through the courtesy of Latham (1973).

Examination of reproductions of three of the lunar seismograms (Toksoz et al 1972) show that the P wave first arrivals can be picked on these seismic records with an uncertainty of about 1/2 second. The S wave arrivals are more difficult to pick. Since I

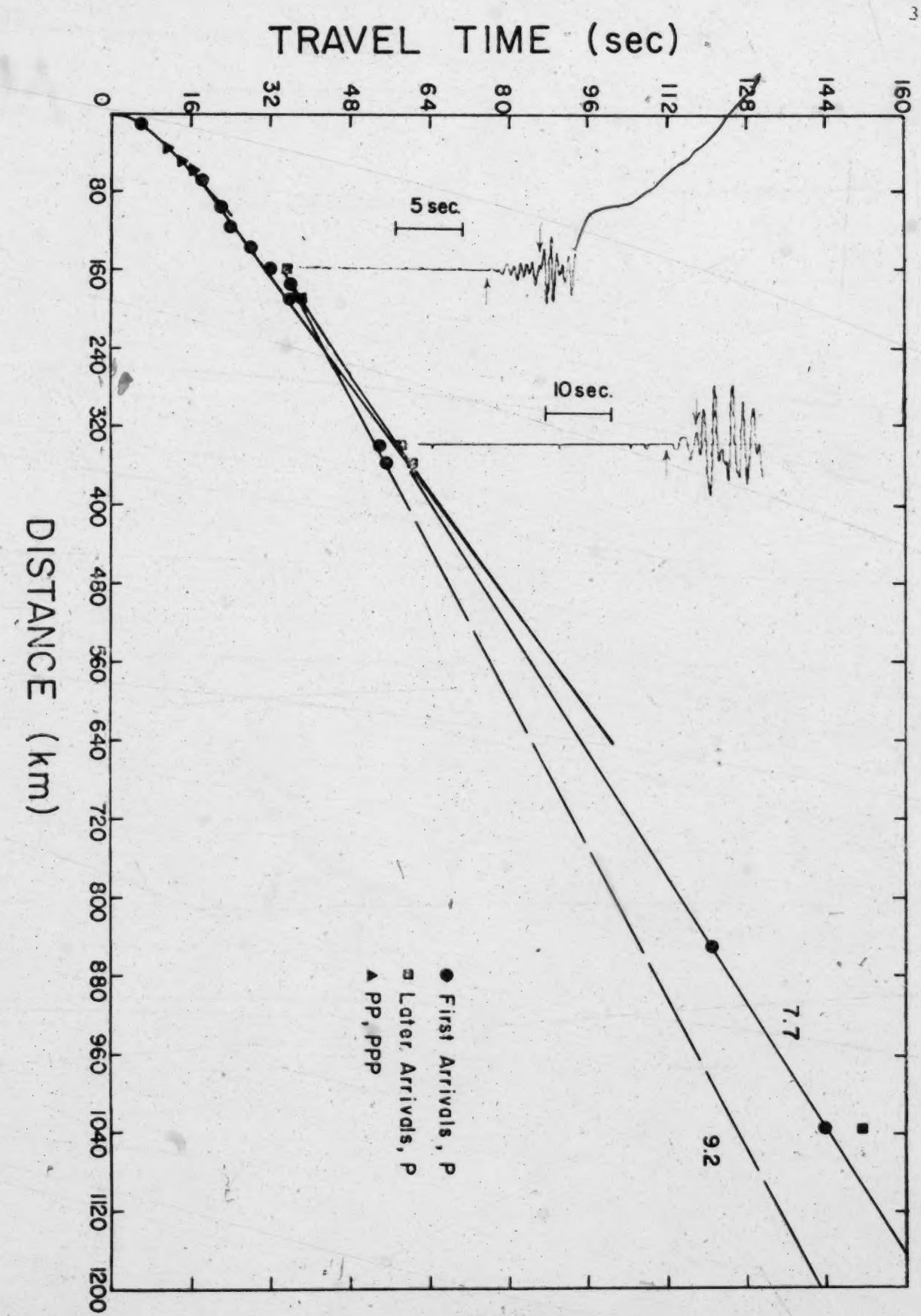

TRAVEL TIME (sec)
0
16
32
48
64
80
96
112
128
144
160
DISTANCE (km)
0
80
160
240
320
400
480
560
640
720
800
880
960
1040
1120
1200
5 sec.
10 sec.
● First Arrivals, P
▫ Later Arrivals, P
▲ PP, PPP
7.7
9.2

FIGURE 6

had access only to scaled down published reproductions of some of the seismograms I assumed that the three seismograms were representative. If this is true, the P wave first arrivals of most of the records could be picked to within an uncertainty of 1/2 second. I assumed that the S arrival uncertainty was about twice this value.

Using a radially symmetric moon small plausible lateral deviations can be taken into account by including them in the observation uncertainty. That is the moon's ellipticity, small lateral variations in velocity structure, and elevation of the source and receiver above the lunar geoid can be included in an uncertainty in arrival time.

A shallow seismic velocity profile obtained from the Apollo 17 active seismic experiment (Kovatch and Watkins 1973) showed that the first few kilometers of the Taurus-Littrow region of the moon have a step-wise velocity depth structure compatible with a series of lava flows. This seismic profile samples both mare and highland regions. Plausible variations in the first few kilometers of the moon composed of a surface layer of such a series of lava flows and/or intruded basalts suggest that differences of up to one second from a standard model can and should be expected. This is the variation in travel time caused by doubling or

Table 1

P WAVE TRAVEL TIMES

Delta in Degrees	Travel Time in Seconds	Weights
.287	5.57	4.0
2.2	17.8	1.75
3.07	22.0	1.75
3.76	25.0	2.0
4.45	28.6	2.0
5.18	32.0	2.0
5.61	35.7	2.0
6.12	26.6	2.0
11.2	57.0	2.0
11.8	55.0	2.0
	61.0	2.0
28.0	123.1	2.0
34.0	155.1*	2.0
	151.0*	1.5

* Only 1 of these 2 arrivals were used. In class B models, which one is used affects the half space velocity acceptance region by ∿.5 km/sec.

S WAVE TRAVEL TIMES

2.2	31.0	1.7
3.76	45.0	1.7
4.45	50.5	1.7
5.61	55.7	1.7
6.12	60.5	1.7

Table 2

P WAVE TRAVEL TIMES†

Delta in Degrees	Travel Time in Seconds	Weights
1.1	10.5	2.0
1.5	14.0	2.0
1.9	17.5	2.0
5.18	35.5	2.0
11.2	54.0	2.0

S WAVE TRAVEL TIMES†

1.1	18.0	1.2
1.5	24.0	1.2
1.9	29.5	1.2
3.07	38.5	1.2
5.61	60.5	1.2
6.12	65.0	1.2
11.8	94.5	1.2
	105.5	1.2

† Note the high uncertainty in these points. It is partially due to their being read off of travel time curves in Toksoz et al (1972) and not from tabulated results.

halving the thickness of a layer in which P wave velocities increase from 1 to 4 km/sec.

As is well known the P and S wave velocity structures are related through Poisson's ratio. I will assume a constant Poisson's ratio equal to .25 and feel that the petrology of the moon is not sufficiently well established to justify the choice of a more detailed Poisson ratio structure. In any case the seismic velocities are not sensitive functions of Poisson's ratio in the range .22 to .28 the experimental ranges observed in lunar rocks. I also feel that knowing the uncertainty in S wave arrivals as poorly as I do that trying to put limits on the Poisson ratio structure would be presumptuous. Thus the S wave arrivals have not been treated independently of the P wave arrivals.

Choice of Inversion Method

The limited amount of seismic data available on the moon from the Apollo seismometers as well as the relative lack of certainty about the internal structure of the moon suggest that an exploration of the range of lunar velocity models implied by the Apollo seismic data might very well be done by means of the Hedgehog technique. The quasi-linear techniques are difficult to apply in the case of travel-time data since incomplete

sets of travel-time data appear not to be well-conditioned to iterative searches for best velocity depth functions. Although it is possible to search the travel-time data by quasi-linear techniques, I prefer to explore the topology of the space of all possible solutions, not just those that can be linearized, for the lunar seismic inverse problem by means of the Hedgehog technique.

The Hedgehog technique is a method for systematically determining all possible combinations of parameters of a class of velocity models which satisfy a set of observed data within given precision limits. I deal here exclusively with P and S travel-times. I do not attempt to interpret the synthetic seismograms, their character has been discussed by Nakamura et al (1970). It is my purpose to determine upper and lower bounds for the velocity structure of the moon. I recognize that interpretations of the synthetic seismograms will refine these upper and lower bounds, particularly in transition zones. It is my purpose here to demonstrate that fairly stringent limits can be derived from travel-time data alone.

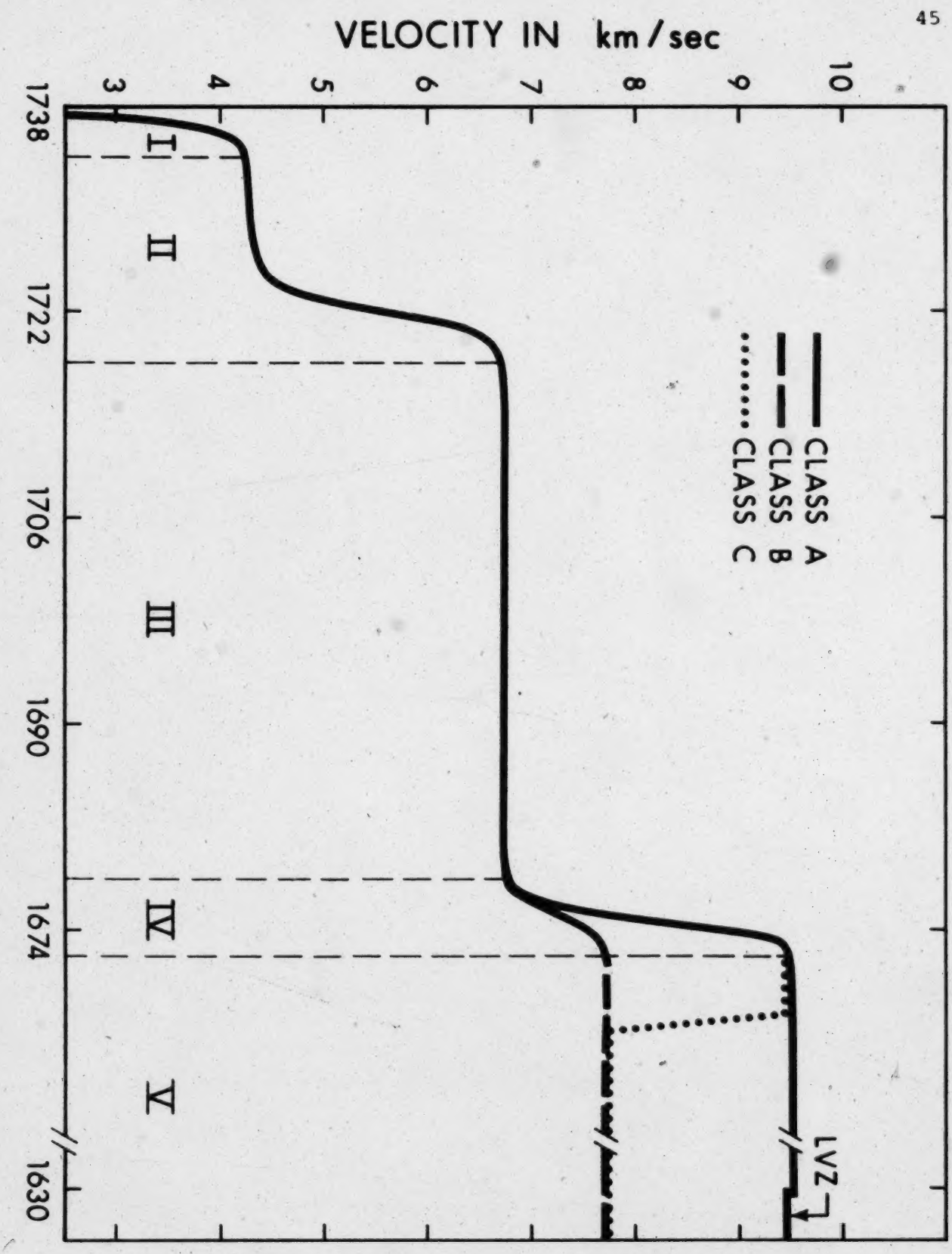
Parameterization and Lunar Models

My Hedgehog program demands that the class of all possible solutions be described in terms of a

limited (less than 16) number of parameters. These parameters can be any number which is plausible, or possibly a number which might by some investigators be considered a rather implausible descriptor of the lunar velocity structure. The conversions from parameters to actual velocity structures need not be linear. There are no practical restrictions on the kind of parameterization or on the kind of conversions that can be carried out between lunar parameters and lunar structure.

The choice of parameters for a Hedgehog program is governed by the need for a compact description of all possible solutions of the inverse problem. Obviously some parameters are not as well suited as other parameters and it is usually not immediately obvious which are the best parameters for a given inaccurate and incomplete set of data. The choice should be made of those parameters which are best determined by the available data.

There is no immediately obvious choice of parameters for the Apollo seismic data. I experimented with a number of sets of parameters which appear to me to be plausible, and chose three sets of parameters which gave compact description of the resulting acceptable lunar models.


VELOCITY IN km/sec
3
4
5
6
7
8
9
10
1738
1722
1706
1690
1674
1630
I
II
III
IV
V
CLASS A
CLASS B
CLASS C
LVZ

FIGURL 7

I choose three classes of models. Each class of models being a separate parameter space differing from the others in its dimension (number of parameters) and in the parameterization (the types of models to be searched). This is due to the possible different interpretations of the data near 1000 km (Table 3). Each class of models was separately inverted by the Hedgehog technique.

Examination of Figure 7 will show the characteristics of the 3 classes of models. Down to region IV they are identical. A description of the models follows:

All Models

Region I - First four kilometers of the moon. No variation allowed in v(r) which is a smooth curve approximating the shallow seismic results of Apollo 17.

Region II - A smoothed step with the position of the top of the step, velocity at the top of the step, and velocity gradient, free parameters for the Hedgehog algorithm.

Region III - A constant velocity layer. This velocity is a free parameter.

Region IV - A smoothed step with the position of the step and the velocity at the top of the step free parameters.

Table 3

P WAVE TRAVEL TIME DATA AT LARGE DELTAS

	Delta in Kilometers*	Travel Time in Seconds	Weight
Meteoroid impact of day 134, 1972	967	131.5	1.0
	1026	138.5	1.0
Apollo 16 SIVB	1099	147.0	0.4
From Table 1	850	123.0	2.0
	1032	151.0	2.0
		155.1	2.0

* 30.3 kilometers equal 1 degree on the lunar surface.

Class A Models

Region V - A constant velocity half space; the velocity in the half space is a free parameter.

Class B Models

Region V - A constant velocity half space with a L.V.Z. at ∿100 km depth; velocity in the half space is a free parameter. Travel-time data in the region 800-1200 km are interpreted as multiples (i.e. PP travel times).

Since there exists a low velocity zone near 100 km in all these models first arrivals near a delta of a 1000 km will not be direct P wave arrivals since this will then be in a shadow for direct P arrivals but will be the multiple PP arrivals.

Class C Models

Region V - A step down to a lower velocity half space; the velocity at the top of the step and the velocity of the half space are free parameters.

Acceptance Criteria

Our criterion for a lunar model are that the following 2 conditions hold for both the P wave and

S wave data separately:

$$(1) \quad \sqrt{\frac{\sum_{i=1}^{N} \left(\frac{O_i - C_i}{W_i}\right)^2}{N}} \leq \sigma$$

$$(2) \quad \text{no more than j of the } \left|\frac{O_i - C_i}{W_i}\right| \geq k$$

where

O_i is the ith observed travel time;

C_i is the ith computed travel time;

W_i is the ith assigned weight;

N is the total number of points on all branches of the travel time curve;

σ is the rejection limit for the r.m.s. deviation;

j,k are rejection limits such that we allow no more than j of our observations to deviate more than a time k.

There exists some difficulty in deciding which observation goes with which calculated phase. The algorithm for choosing phases to which data is to be matched was developed at the University of California, Los Angeles in 1967 (Keilis-Borok 1971). In the absence of $dT/d\Delta$ measurements, it consists of determining that theoretical phase which is closest, in an absolute sense, to the observed phase. All theoretical phases are computed for a given lunar model before the

comparison is made, those phases which follow or precede other phases by less than 0.5S were discarded. Since the theoretical travel-time computation uses power law interpolation, it should be observed that the amplitude calculations may have spikes at critical deltas. The algorithm for discarding unreasonably close arrivals uses amplitudes. I have checked that these amplitudes were calculated in ranges of Δ in which our calculation can be expected to be reasonably accurate.

It should be noted that the relative weights W_i have been semi-quantitatively assigned to the observations. That is a qualitative rather than quantitative analysis of the quality of arrivals at differing deltas and of various phases has been attempted. I have assigned various weights to these values with the weights dependent upon both comments and assigned uncertainties in Latham et al (1972), Toksoz et al (1972) and Latham (1973). I have also separated the data into three separate tables.

The first table consists of first and second arrivals with no possible argument over the identification of the arrivals. That is there is complete agreement between the various groups analyzing the data in the picking of the seismograms for these arrivals.

The second table consists of possible multiples such as PP or SSS and some poor quality first and second arrivals. These multiples are in some cases hard to pick and there exists some ambiguity in the picking of the first and second arrivals. That is there is some argument whether these can be observed on the seismograms.

The third table consists of data near 1000 km including data from Table 1 and arrivals from the impacts of a meteoroid and the Apollo 16 Saturn IVB. Positions and impact times of these latter two impacts were calculated from data not present in these tables by Latham et al (1972). Thus these latter two points on the travel time curve have a greater uncertainty and are in a weak sense dependent upon a lunar model to about 60 km depth.

Due to the differing acceptability of the data in these tables several different cases had to be considered in the Hedgehoging of the data.

Parameter Step Size

I ran several groups of Hedgehogs on the data with both a coarse grid - one where the various velocity step sizes were 1 km/sec and position step sizes were 10 km - and a fine grid - one where the various step sizes were 1/2 that of the coarse grid.

Hedgehog Runs and Results

I ran several Hedgehogs on each of these models. The results are summarized below.

1) Class A Models:

I could not find any acceptable models of this type with any reasonable set of acceptance limits. That is the data appears internally contradictory in that the data points going up to make the triplication (either from Table 1 or 2) suggest a half space velocity of 9.5 km/sec while the travel times near deltas of 30° suggest a velocity of 7.75 km/sec. With this class of model I could not reconcile these seemingly contradictory observations.

2) Class B Models:

These models are characterized by a poorly resolvable L.V.Z. near 100 km depth. With an L.V.Z. located at that depth I have no direct P arrivals near a delta of 30°. Although I am in a shadow zone, multiples (PP for example) may be possible. Interpreting the arrivals near 1000 km as PP the following results were obtained:

i) Using Table 1 data; and the acceptance criterion that $\sigma = 2.8$, $j = 0$, $k = 1.2$ and a coarse grained search I obtained the results in Figure 8.

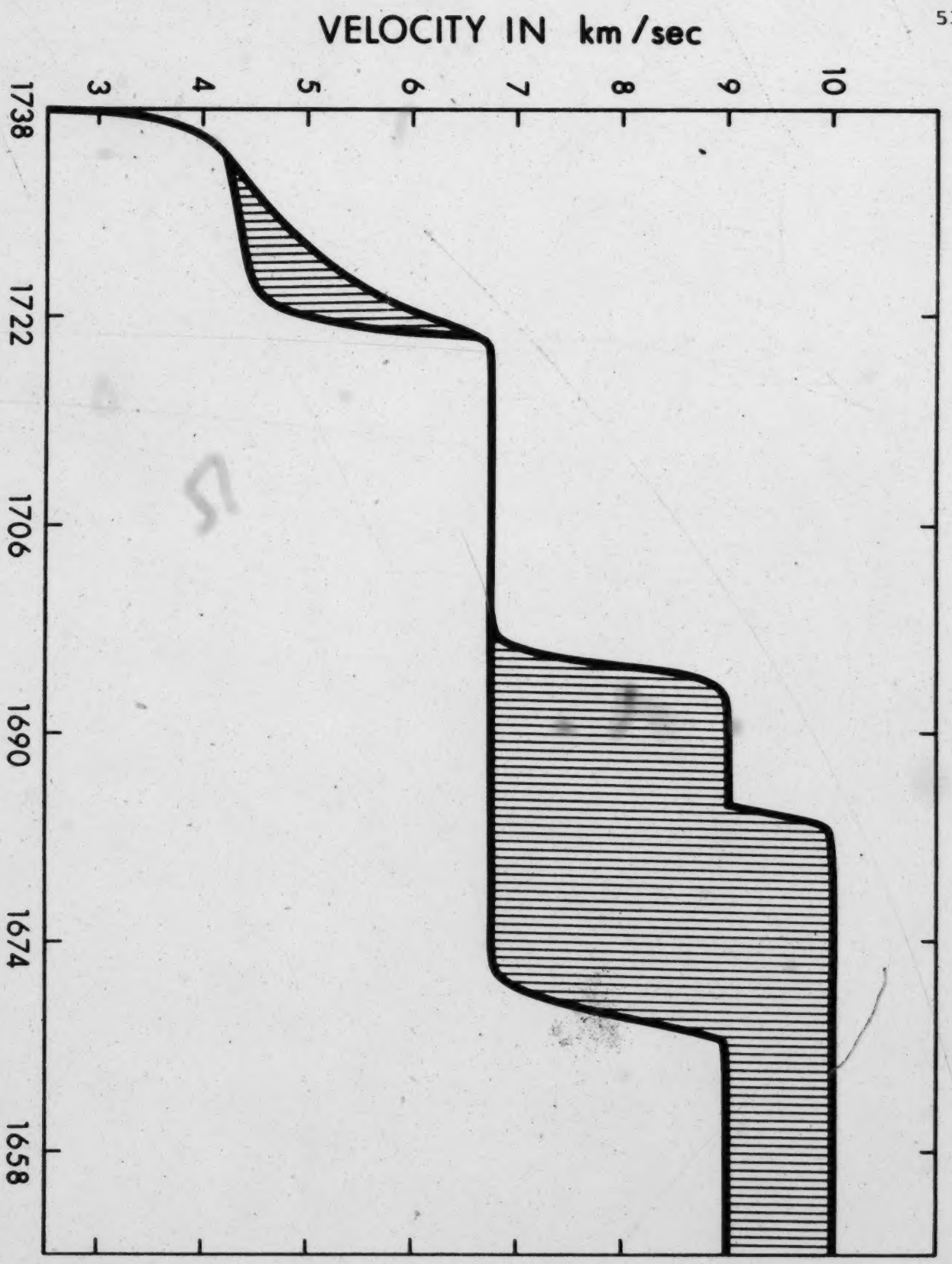


FIGURE 8

ii) Using Table 1 and 2 data; $\sigma = 6.0$, $j = 0$, $k = 2.4$, and a fine search I obtained Figure 9.

iii) Using Table 1,2,3 data - no acceptable solutions were found.

I see that in cases i) and ii) the acceptable models are essentially the same. I can see that this is so because there are fewer but more accurate data points in case i) than in case ii). This shows up in the greater uncertainty of the location of the layer IV interface in case ii) due to a larger choice of acceptance limits σ and k. There exists one possible difference in these solutions in that there can exist possible differences in velocity gradient in region II.

In these two cases I have explored the possibility that the uncertainties (σ and k) should be reduced to 1/2 of the listed values in cases i) and ii). We then found no acceptable solutions. Increasing the uncertainty obviously increased the number of possible solutions. Increasing the number of possibly badly identified phases (j) from zero to two caused a spectacular increase in the number of solutions. A badly identified phase is an arrival which deviates from its matching theoretical arrival by more than an acceptable uncertainty k (equation 2).

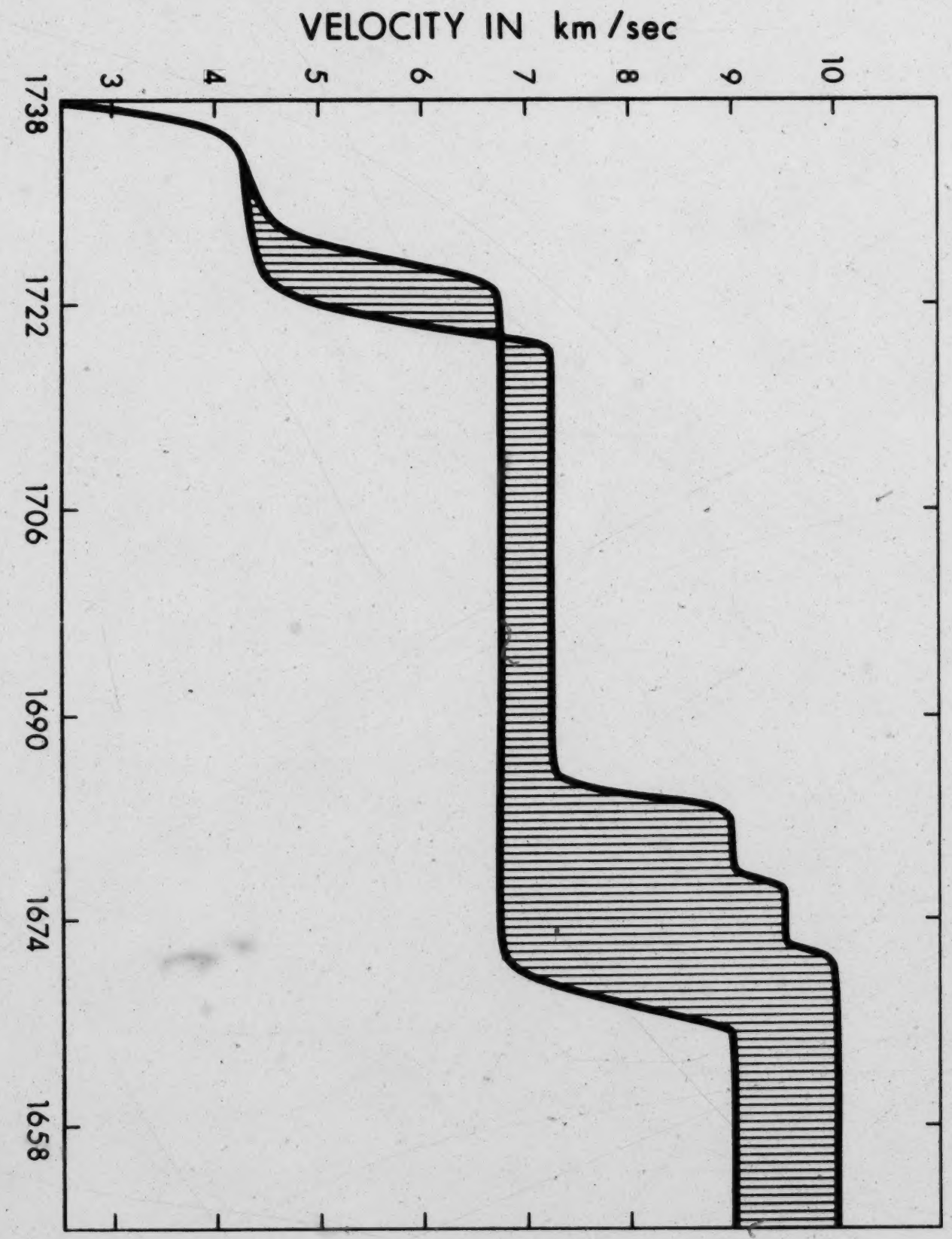


FIGURE 9

If all the travel times near 1000 km (Figure 4) are graphed then it is obvious why there are no acceptable solutions in case iii) since the data appears to be inconsistent.

I can conclude the following. IF the Table 3 data are in error ≥10 sec, the models in cases i) or ii) are correct. IF the Table 3 data are more accurate than this I must conclude that my model is inadequate. For the case that Class B Models and Figure 8 are accurate descriptors of the seismic structure in the moon the high velocities in the half space should be noted since such high velocities raise certain strict petrological constraints (Toksoz 1972).

3) Class C Models:

These models are characterized by a thin high velocity lid overlying a low velocity zone. Classically this would produce a shadow zone from 12° to 50°. Applying wave theory in this classical shadow zone I suspect the following:

i) A head wave travelling along the top of the high velocity lid propagating with the velocity of the lid.

ii) Possible leakage of the low frequency components of this headwave into the L.V.Z. Thus I will have a high pass filtered headwave. The amplitude of the headwave will be highly variable due to the nature of

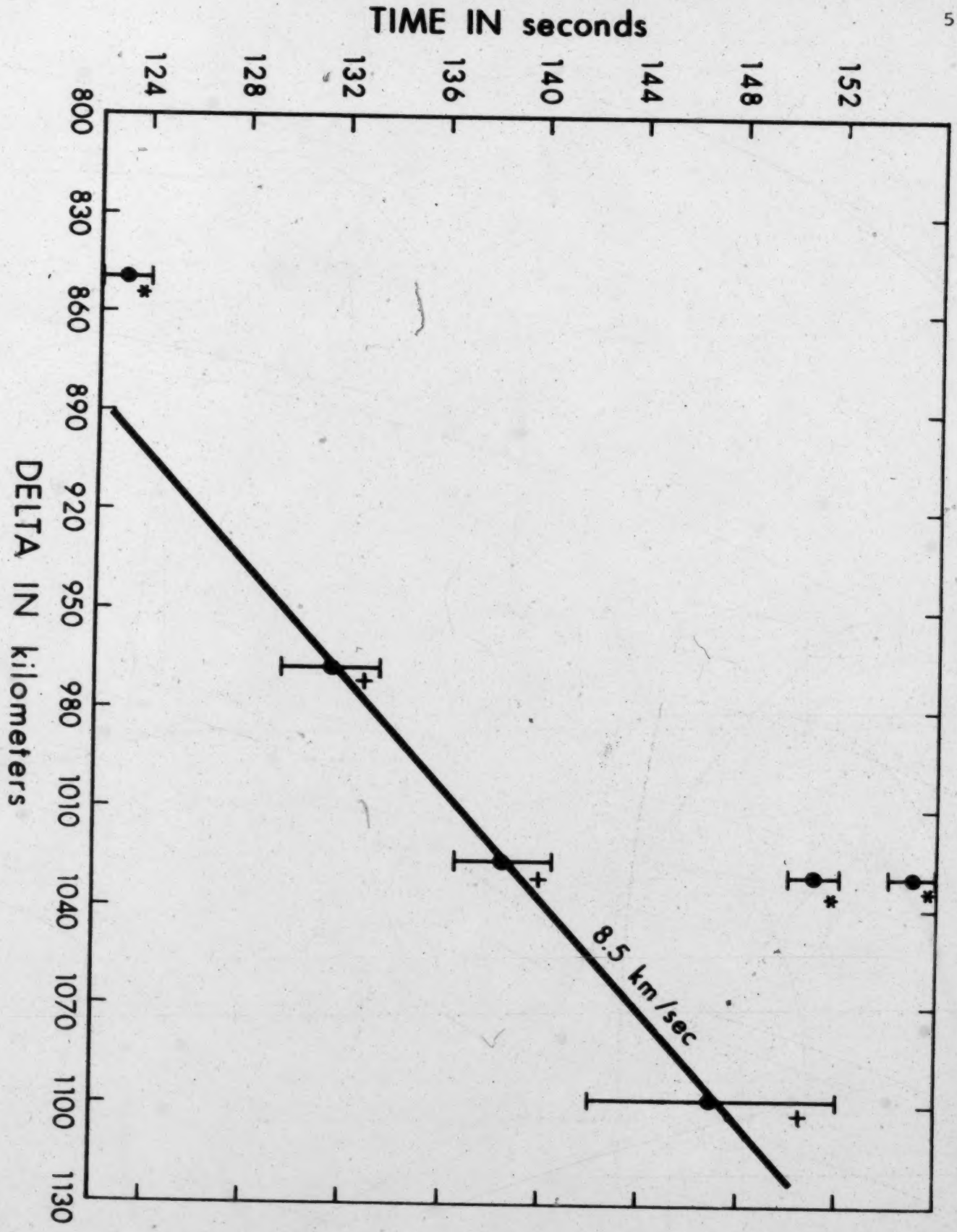

TIME IN seconds
124
128
132
136
140
144
148
152
800
830
860
890
920
950
980
1010
1040
1070
1100
1130
DELTA IN kilometers
8.5 km/sec

FIGURE 10

headwaves in that small changes in either gradient of velocity in the lid, or curvature of the lid interface will cause drastic changes in amplitude.

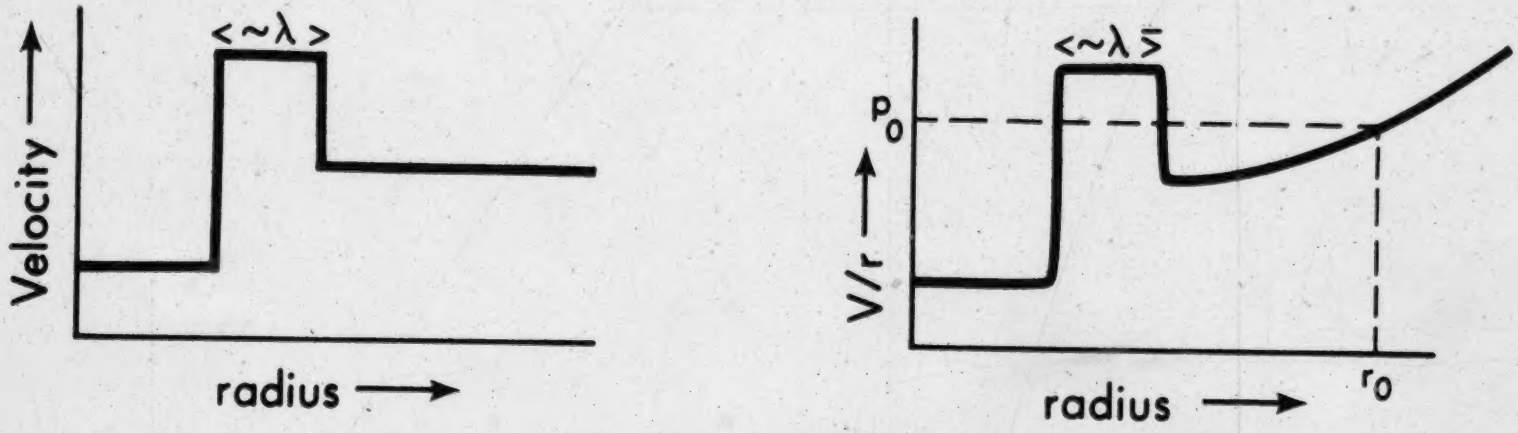
iii) A tunneled wave having tunneled into and cut of the L.V.Z. The tunneling amplitude is a function of frequency, velocity contrast between top of the lid and L.V.Z., width of the lid, and of delta. For a thin lid ~1 seismic wavelength this tunneling amplitude is relatively large. This tunneled phase will be effectively low pass filtered (Figure 11).

Unfortunately the travel-time program that we have in our Hedgehog package does not take into account either tunneled phases or headwaves. Yet if we make the interpretation that the data in Figure 10 can be analyzed as follows:

i) two of the three phases marked * are tunneled phases (I can ignore either of the points at 1038 km);

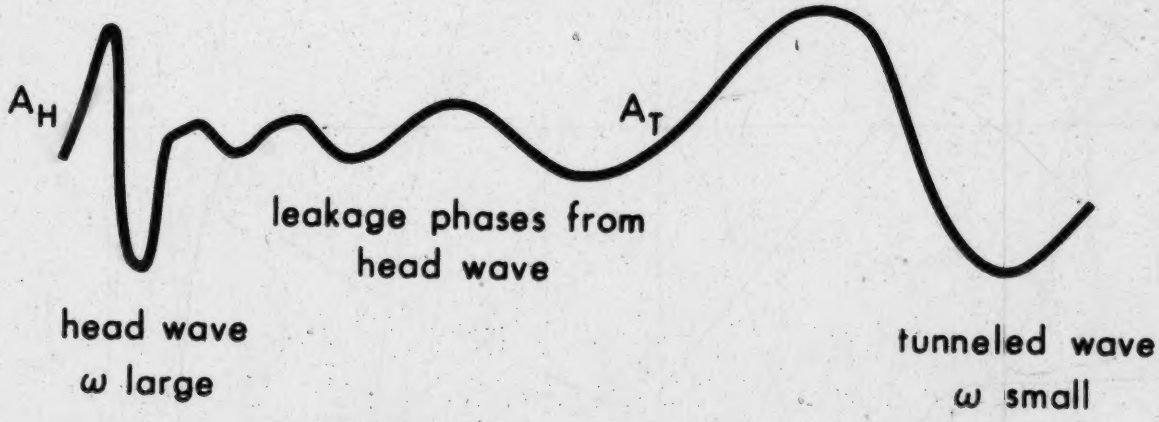
ii) the three phases marked † are headwaves;

then this is consistent with a thin high velocity lid. The position of the lid (and the velocity structure above the lid) is that of the interface in the Class B Models. The velocity at the top of the lid is



A wave incident at the high velocity lid with ray parameter p_0 will tunnel energy through the lid which will be reflected at r_0. Some of this energy will resonate in the L.V.Z. the rest will tunnel out.

FIGURE 11



APPROXIMATE SEISMOGRAM

$\sim$8.5 km/sec (between 8.0 and 9.5 km/sec) and a constant velocity half space of $\sim$7.5 km/sec (7.25 to 7.75 km/sec).

If this last class of model represents the true moon then a spectral analysis of seismograms in the range 500 km $\leq \Delta \leq$ 1500 km may show evidence of such a lid.

Conclusions

Down to a radius of $\sim$1670 km all 3 classes of models coincide. The data gives moderately good resolution of the model down to this depth and we are fairly certain that the seismic velocity profile lies somewhere in or very near to that of Figure 2. The width of the acceptance region is highly dependent upon the acceptance criterion. We cannot tell how good these are without a careful and critical analysis of the seismograms.

Below this depth I am faced with the following prospects:

1) Class A Models:

   These appear too simple.

2) Class B Models:

   For these to be acceptable some of the data points near Δ of 1000 km must be incorrect.

3) Class C Models:

These agree well with all the seismic observations but the petrology of a high velocity lid followed by a L.V.Z. on the moon requires special, though not necessarily uncommon, petrological constraints.

Relation to Lunar Petrology

The relation of seismic velocity structure to lunar petrology has been discussed in both Toksoz et al (1972) and Latham et al (1972). To summarize their conclusions for my class two and three models I have made the following correlations using their results.

Region I probably consists of a series of dry basaltic lava flows (not a lunar soil as was thought prior to the Apollo 17 landing) grading into the second region which has velocities corresponding to samples of lunar basalts that have been examined. The third region may be a mixture of gabbros, anorthosites and pyroxenites.

In the case of Class B Models the half space (region V) velocity is too high except for mineral assemblages such as pure olivine. But if I invoke a thick olivine layer - the half space - the density of olivine is so high that the mass of a moon with such an inner structure would be completely in

disagreement with known values derived from astronomy. There have been no lunar samples returned with a velocity greater than 7 km./sec.. This could be due in part to the rarity of such outcrops on the lunar surface. There is a similar example of the occurrence of dunite on the earth.

In the case of class C models the half space velocity could correspond to pyroxenes and/or eclogites.with possibly some small amounts of olivines. The high velocity lid over this layer could be a thin high density garnet layer which Anderson (]973) predicts would be stable near this depth in the lunar interior.

leaf 63 omitted in page numbering

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The entire Hedgehog Package is a superstructure erected about a systematic search routine in an N-dimensional space. This routine was first used successfully in a geophysical context by V.P. Valus (1968) in Moscow, and was subsequently used as a basis for the present package which evolved at U.C.L.A. under the guidance of L. Knopoff and V.I. Keilis-Borok. The present version incorporating a number of small modifications is based on the version KBPRGI implemented at Edmonton by E. Nyland, and E. Roebroek. The description that follows is based on the program KBCAM currently available on the IBM 370 at the University of Cambridge.

Some features of the current University of Alberta version are not in KBCAM, and vice-versa. The user should beware. A quick look at the source will be profitable.

The University of Alberta version has a capacity for interactive graphics using routines CUCGRP, GRID, hardware, and software. For more details refer to the CUCGRP writeup, and "Computer Graphics' for Application Programmers".

1.1 THE HEDGEHOG TECHNIQUE

This section attempts to describe briefly the

theoretical background to the Hedgehog technique- this is described in much more detail in section 5, and anyone proposing to use this package seriously should read that section carefully. If we consider an Earth model, we may define this in terms of a number of parameters - e.g, we may choose seismic velocities, densities, Q, electrical, and thermal conductivities, etc. The particular choice of parameters taken depends on the particular type of problems being considered. In this Hedgehog scheme we look for the solution of an inverse problem not merely as a set of velocity-depth, and density-depth functions but as a set of suitable Earth parameters. Those parameters which may be the most useful in a given inverse problem are not necessarily the obvious ones mentioned above - some combination of these may be more appropriate.

It should be emphasized that the fundamental elements of a Hedgehog solution are parameters whose choice is up to the user. It is these parameters which are perturbed to generate a suite of structures, and the method of perturbance gives rise to the Hedgehog.

Within the Hedgehog method we allow a limited number ($N<16$) of the parameters to vary, and search for those combinations of parameters which give an Earth structure whose properties agree with the observations within the required precision. We have thus a problem, how do we

generate an Earth structure from a given set of parameters. This is acheived within this version by the use of the symbolic language PEAR.

As an example of the possible problems note that alpha can be derived from the density by an equation of state, and is related to beta by poissons ratio which may be a function of depth.

For each of the N variable parameters we specify an upper and a lower bound and a step size. This defines a region of search in parameter space spanned by a N-dimensional network in which each node is associated with a different Earth structure. By some means, e.g. guess-work or Monte-Carlo methods, we choose on node as a starting point for the Hedgehog. This node may be 'good' (i.e. the corresponding Earth structure fits the data) or, it may be 'bad'. The method is to search all nodes in the vicinity of the initial node and to determine the region in the N-dimensional space which contains good nodes. We do this by changing each of $m<N$ parameters one step in the positive direction and one step in the negative direction across the network until all the neighbours of this first base point have been investigated. Since we move only one step on any parameter the order of the Hedgehog, m, is the numbers of parameters varied simultaneously. Now, hopefully, some

of the neighbours are 'good' nodes. These we record by their node indices and when all the neighbours are tested we use the next good node as a base (nb an order of 1 may miss a point). The process is illustrated by figure 5 Why the name Hedgehog ? - well the pattern of movements in parameter space is sufficiently spiny to suggest to some the shape of a Hedgehog. It is possible that the zone of good points is not simply connected or consists of distinct regions - the latter can be handled by using the combination of Monte-Carlo, and Hedgehog method available in this form of the Package. In the former case apparent fragmentation of the volume of 'good' nodes may occur if the steplength is not small enough.

1.2 USES OF THE PACKAGE

By suitable choice of Program Procedure and calculation limits the package may be used for the following purposes:

a) Inversion package for Love and Rayleigh wave dispersion and Q and SH travel times - these may be inverted separately or jointly

B) Dispersion package - calculation of Love and Rayleigh wave dispersion and Q

C) Sh and P travel times - calculation of theoretical travel

times for various phases as well as ray parameters

The following notes should enable one to use the package but serious users should make themselves familiar with the source program.

2 ORGANIZATION OF THE PACKAGE

The basic structure of the Package breaks down into 6 component parts

a) Input and Control section

b) Choice of a parametrised Model

c) Calculation of an Earth structure form the parameters

d) Surface wave routines

e) Travel time routines

f) Acceptance routine

The package is well documented through comment cards throughout particularly in the introduction, which is reproduced for reference in Appendix 1. It is recommended that this should be read before any attempt

is made to use the package. The following descriptions should also be supplemented by reading the source.

2.1 INPUT AND CONTROL SECTION

This consists of the routines

BLOCKDATA

MAIN

INPUT

BLOCK DATA: This initialises sizes of various arrays used in the package and set up default options on the handling of input and output

Main: The function of the main program is to act as a (Grafic) branching control to the various facilities in the package. The possibilities are listed below.

```
C
C  SELECT PATH ACCORDING TO INDEX
C
C  BRANCH TABLE
C
C    1    TERMINATE PROCESSING OF ONE BATCH AND GET NEW
DATA
C    2    HEDGEHOG PROCEDURE
C    3    COMPUTE CROSS SECTION.  CHECK AGAINST LIMITS
C    4    COMPUTE LOVE DISPERSION CURVE
C    5    COMPUTE RAYLEIGH DISPERSION CURVE
C    6    COUNT ITERATIONS.  TERMINATE IF TOO MANY.
C    7    ACCEPT POINT
C    8    REJECT POINT
C    9    SET UP PROGRAM PARAMETERS
```

```
C  10     CONVERT RADIAL STRUCTURE TO LOVE LAYER STRUCTURE
C  11     CONVERT RADIAL STRUCTURE TO LOVE AND RAYLEIGH
C                      LAYER STRUCTURE
C  12     CONVERT RADIAL STRUCTURE RAYLEIGH LAYER
STRUCTURE
C  13     COMPUTE TRAVEL TIME FOR Sh
C  14     RESTART PROGRAM
C  15     EXCUTE CONSOLE DISPLAYS
C
C  16     * MONTE-CARLO PROCEDURE
C  17     * COMPUTE LOVE WAVE DERIVATIVES
C  18     * COMPUTE RAYLEIGH WAVE DERIVATIVES
C  19     * COMPUTE Q FOR LOVE WAVES
C  20     * COMPUTE Q FOR RAYLEIGH WAVES
C  21     * COMPUTE LOVE WAVE DISPERSION CURVE BY
DERIVATIVES
C  22     * COMPUTE RAYLEIGH DISPERSION CURVE BY
DERIVATIVES
C
C  29     + CALCULATE GRAVITY FIT
C  30     + COMPUTE TRAVEL TIME FOR P
C
```

The control of the branching is acheived by the contents of the array CNTL which is set by the program procedure. In section 3 we describe how to set up such a Program Procedure for the package.

INPUT: As it's name suggests this routine is the general input facility to the package. Control within the input routine is acheived by a transfer table listed below

```
C
C  BRANCH TO READ GROUP
```

* A feature of KRCAM not implemented at University of Alberta
\+ A feature under development

```
C
C  TRANSFER TABLE
C
C  L =  1  PARAMETERS
C       2  CONLIM  LIMIT PARAMETERS FOR VARIOUS
CALCULATIONS
C       3  CROSS SECTION DATA
C       4  CLEAR PROGRAM
C       5  LOVE DATA
C       6  RAYLEIGH DATA
C       7  BODY WAVE STATION DATA
C          S ARRIVAL TIMES
C       8  PROGRAM PROCEDURE
C       9  GRADIENT LIMITS
C      10  END OF DATA
C      11  PNTSBL  RECOVER GOOD POINTS ALREADY FOUND
C      12  STOP    TERMINATE EXECUTION OF THE PROGRAM
C      13  SYSIN   ALTER INPUT DATA SOURCES
C      14  SYSOUT  ALTER OUTPUT DATA GROUPS
C      15  IOALG   CHANGE READ-IN ALGORITHM FOR DATA
C      16  OUTFLG  CONTROL OUTPUT OF DATA
C      17  * SURFACE WAVE Q DATA
C      18  WRITE COMMENT CARD
C
C      19  LOAD    P TRAVEL TIME DATA
C      20  + GRAVITY DATA LOAD
C      21  ICNLIM  PARAMETERS READ IN
C
```

The control of the branching here is acheived through the first four letters of a control word which are compared with values held in the array KEYTBL. These are in fact the first four letters of the descriptions above except for 17: QDAT, 18: COMM, and 19: P BO. By this means the required data may be read into the appropriate groups of variables.

2.2 CHOICE OF A PARAMETRISED MODEL

* A feature of KBCAM not implemented at University of Alberta
+ A feature under development

This consists of two routines

* MCSET

HGEHOG

* MCSET: This is a routine to choose points and nodes in parameter space by the use of the Monte-Carlo method.

There are five possible schemes determined by the setting of the flag MCALG.

MCALG = 1 Monte - Carlo a good point

= 2 Monte - Carlo for a good node

= 3 Monte - Carlo for a good point then move to nearest node and use the Hedgehog technique

= 4 Monte - Carlo for a good point and then to Hedgehog only if the nearest node is good

= 5 Monte - Carlo for a good node then move to Hedgehog

except in cases 1,2 it will be noted that the routine is intended to choose a starting point for the Hedgehog technique to enable searching of multiple regions to be acheived a list is built into HEGEHOG

(entry NODCHK) to see if a Monte - Carlo node has
been looked at before.

HGEHOG: This is the basic Hedgehog routine which generates a
sequence of nodes in the parameter space to be used to
produce Earth structures. This routine has three entry
points HGEHOG, PNTPCK, and RESTRT.

PNTPCK: This stores acceptable nodes in an array.

RESTRT: This can be used to enter a set of acceptable node for restarting an unfinished run.

2.3 CALCULATION AN EARTH STRUCTURE

This also consists of two routines
CRSCHK
LVCVT

CRSCHK: This routine uses the input from the symbolic language PEAP (see section 4), and constructs a cross
section i.e. an Earth structure corresponding to

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† A feature under development

a

given set of parameters, and checks that this structure lies within the limits (upper and lower bounds, and gradient limits) specified in the input

LVCVT: This is a conversion routine using Earth flattening techniques to convert radial Earth models to Love and Rayleigh flat layer models. For Love waves the exact method of Biswas and Knopoff is used for Rayleigh waves the empirical method of Polt and Dorman is used

2.4 SURFACE WAVE ROUTINES

There are five routines in this section

TSW

SWEVAL

SWRAYL

SWLOVE

* SWDER

TSW: This acts as a control monitor on the handling of the othersurface wave routines which are accessed from

MAIN

through TSW.

SWEVAL: This tests a theoretical dispersion curve against a

data set, and produces a flag RTCNTL.

RTCNTL = 0 if structure is good

= 1 if structure is bad

SWPAYL: Fast Rayleigh wave dispersion program - Knopoff's method

SWLOVE: Fast Love wave dispersion program - Thomson-Haskell method

* SWDER: This is a multipurpose routine which calculates the

structure derivatives of the dispersion curves for a

Particular model - these may then be used either to

calculate dispersion curves for close structures or to

calculate surface wave 1/Q by Anderson's method

Each of the routines SWPAYL, SWLOVE, SWDER calls SWEVAL

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+ A feature under development

so the the RTCNTL flag is set after entering any of these routines.

2.5 TRAVEL TIME ROUTINES

This section has ten routines

TST	TPT
DELFS	DELFP
DELRS	DELRP
DCORRS	DCORRP
TSEVAL	TPEVAL

TST/TPT: These routines calculate the travel times, ray parameters, and amplitudes for each phase arriving a given distance. Mode conversion is neglected so that these are strictly for SH/P waves. These routines call TSEVAL/TPEVAL, and so set a RTCNTL flag.

DELFS/DELFP: These calculate delta given the ray parameter.

DELRS/DELRP:

DCORRS/DCORRP: These provide a depth correction to delta for

a given ray parameter.

TSEVAL/TPEVAL: These complicated routines compare theoretical,

and observed phases at each station, and determine whether the fit to the data is good,

producing a flag RTCNTL.

PTCNTL = 0 if structure is good

= 1 if structure is bad

Note the use of ICNLIM (see the comments in BLKDTA).

2.6 ACCEPTANCE ROUTINE

ACCEPT: This indicates the production of a suitable parameterised earth structure.

2.7 + GRVTST

This calculates the gravity profile from a density model.

3 THE PROGRAM PROCEDURE

3.1 DESCRIPTION OF THE PROGRAM PROCEDURE

The program procedure which controls the sequence of operations in the running of the package consists of a

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+ A feature under development

sequence of integers, each having a definite location in the sequence e.g.

	9	3	6	10	4	100	129	17 .....
Locn.	1	2	3	4	5	6	7	8 .....

The instruction integers j have the following significance

j< 100 - Branch to label j*1000 within Main

j> 100 - Jump to location (j-100) in the program procedure and perform the instruction located there

Following a branch to a routine which calls upon a evaluation, and therefore sets an RTCNTL flag. There are two conditional instructions depending on whether the test was good or bad. For example consider

4 108 128

4 - Love wave dispersion calculation and testing

108 - RTCNTL=0, structure good GO TO INSTRUCTION 8

128 - RTCNTL=1, structure bad GO TO INSTRUCTION 28

The branching table for the instructions j is

```
C
C  SELECT PATH ACCORDING TO INDEX
C
C  BRANCH TABLE
C
C   1    TERMINATE PROCESSING OF ONE BATCH AND GET NEW
DATA
C   2    HEDGEHOG PROCEDURE
C   3    COMPUTE CROSS SECTION.  CHECK AGAINST LIMITS
C   4    COMPUTE LOVE DISPERSION CURVE
C   5    COMPUTE RAYLEIGH DISPERSION CURVE
C   6    COUNT ITERATIONS.  TERMINATE IF TOO MANY.
```

```
C   7     ACCEPT POINT
C   8     REJECT POINT
C   9     SET UP PROGRAM PARAMETERS
C  10     CONVERT RADIAL STRUCTURE TO LOVE LAYER STRUCTURE
C  11     CONVERT RADIAL STRUCTURE TO LOVE AND RAYLEIGH
C                    LAYER STRUCTURE
C  12     CONVERTS RADIAL STRUCTURE RAYLEIGH LAYER
STRUCTURE
C  13     COMPUTES TRAVEL TIME
C  14     RESTARTS PROGRAM
C  15     EXECUTE CONSOLE DISPLAYS
C
C  16     * MONTE-CARLO PROCEDURE
C  17     * COMPUTE LOVE WAVE DERIVATIVES
C  18     * COMPUTE RAYLEIGH WAVE DERIVATIVES
C  19     * COMPUTE Q FOR LOVE WAVES
C  20     * COMPUTE Q FOR RAYLEIGH WAVES
C  21     * COMPUTE LOVE WAVE DISPERSION CURVE BY
DERIVATIVES
C  22     * COMPUTE RAYLEIGH DISPERSION CURVE BY
DERIVATIVES
C
C
C  29     + CALCULATE GRAVITY FIT
C  30     + COMPUTE TRAVEL TIME FOR P
C
```

3.2 EXAMPLE OF A PROCEDURE

As an example of a Program Procedure we give below and annotated version of a procedure which uses many of the features of the package (We will use the shorthand "In" for the instruction n)

```
INSTRUCTION

 1       9      set up program parameters

 2       3      COMPUTE A CROSS-SECTION FROM PARAMETERS

 3       6      COUNT NUMBER OF ITERATIONS, TERMINATE IF
TOO MANY
```

* A feature of KBCAM not implemented at University of Alberta
\+ A feature under development

```
 4      10      CONVERT A RADIAL TO A LOVE WAVE STRUCTURE
 5       4      COMPUTE LOVE WAVE DISPERSION CURVE AND
CHECK
                AGAINST DATA
 6     108      RTCNTL = 0, GO TO I8
 7     126      RTCNTL = 1, GO TO I26 'reject'
 8      12      CONVERT A RADIAL TO RAYLEIGH WAVE
STRUCTURE
 9       5      COMPUTE AND CHECK RAYLEIGH WAVE DISPERSION
10     112      RTCNTL = 0  GO TO I12
11     126      RTCNTL = 1  GO TO I26 'reject'
12      13      TRAVEL TIME ROUTINE
13     130      RTCNTL = 0  GO TO I30 'provisional accept'
14     126      RTCNTL = 1  GO TO I26 'reject'

15       3      COMPUTE CROSS-SECTION FROM PARAMETERS
16       6      COUNT ITERATIONS TERMINATE IF TOO MANY
17      10      CONVERT RADIAL TO LOVE STRUCTURE
18      21      COMPUTE LOVE DISPERSION BY DERIVATIVES
                AND CHECK AGAINST DATA
19     121      RTCNTL = 0  GO TO I21
20     126      RTCNTL = 1  GO TO I26 'reject'
21      12      CONVERT RADIAL TO RAYLEIGH STRUCTURE
22      22      COMPUTE RAYLEIGH WAVE DISPERSION BY
DERIVATIVES
                AND CHECK AGAINST DATA
```

```
23    112      RTCNTL = 0  GO TO I12 'travel times'
24    126      RTCNTL = 1  GO TO I26 'reject'
25      0        reject sequence
26      8      REJECT ROUTINE
27      2      OBTAIN NEW PARAMETERS FROM FROM HEDGEHOG
28    102      GO TO I2 're-iterate'
29      0        derivative calculation
30     17      CALCULATE LOVE WAVE DERIVATIVES FOR A GOOD
MODEL
31    133      RTCNTL = 0  GO TO I33
32    126      RTCNTL = 1  GO TO I26 'reject'
33     18      CALCULATE RAYLEIGH DERIVATIVES FOR A GOOD
MODEL
34    137      RTCNTL = 0  GO TO I37 'accept as a good
model'
35    126      RTCNTL = 1  GO TO I26 'reject'

36      0        accept routine
37      7      ACCEPT ROUTINE
38      2      OBTAIN NEW PARAMETERS FROM HEDGEHOG
39    115      GO TO I15 're-iterate'
```

The purpose of this procedure is that having once found a good structure, computing time should be saved by using derivatives calculated for this structure to look at nearby structures. As soon as a 'bad' structure is

found we return to full calculation as written, if during calculation of the derivatives a bad structure flag is obtained the previous 'good' point is rejected on the grounds that it is only acceptable

4 SETTING UP A STRUCTURE USING PEAR

The purpose of the coding language PEAR (Parameterised EARth models) is to enable a user to specify a model in terms of parameters he wishes and then to be able to deduce Earth structures of a form acceptable for the evaluation routines in the rest of the package.

The way this is acheived is to introduce a set of 256 notional registers into the package, and to use these to compute from the parameter array (XCHH), generated by the Monte - Carlo or Hedgehog procedures, values to place in the Earth structure array (EARTH). The operations on these registers are carried out using PEAR code.

PEAR is probably more than adequate to introduce any plausible constraints into the Inversion procedure.

Particularly one may specify a relation between alpha and beta which depends on the layer number, one can introduce an equation of state such as the Murnaghan equation or, one may study quite complicated "parameters" such as the surface wave impedance.

4.1 THE LANGUAGE PEAR

There are 29 instructions in this code, each being a 4 letter mnemonic for a certain set of operations. These are:

CLAE, CLAH, CLAC, CLAR

ADDE, ADDH, ADDC, ADDR

SUBE, SUBH, SUBC, SUBR

MLTE, MLTH, MLTC, MLTR

DIVE, DIVH, DIVC, DIVR

EXPE, EXPH, EXPC, EXPR

STRE, STRT

IPTE, IPTR

END,

Each instruction card has the following format

A4	I10	I10	I10	F10.5	F10.5
INSTR	J	N1	N2	T1	T2

The various fields are handled as follows

INSTR: The coded instruction INSTR is always present - the number of the others depends on the operation

J: Associated with this system we use a set of 256 registers and the index J always refers to the current register

Each instruction operates in terms of a Variable Var defined in terms of N1, N2, T1, T2 as follows

Class E commands (XXXE):

Here N1, N2 are the indices of the location in the array EARTH

VAR = EARTH(N1,N2)

Class H commands (XXXH):

Here N1 is the index of the location in the array XCHH

VAR = XCHH(N1)

Class C commands (XXXC):

VAR = T1

Class R commands (XXXR):

Here VAR is the contents of register N1

For each of the classes there are 5 types of command

CLAX: Clear register J and add Var

ADDX: Add VAR to the contents of register J (CJ)

SUBX: Subtract VAR from CJ

MLTX: Multiply CJ by VAR

DIVX: Divide CJ by VAR

EXPX: Exponentiate CJ by VAR

There are in addition 4 assignment instructions

STRE: Store CJ in EARTH(N1,N2)

STRT: Store CJ in EARTH(N1,N2) but raise an error flag if T1 > CJ, or T2 < CJ

IPTE: Put T1 into EARTH(N1,N2)

IPTR: Put T1 into register J

The processing of the PEAR instructions is terminated by the instruction END.

4.2 AN EXAMPLE OF THE USE OF PEAR

In the subsection CROSS-SECTION DATA of the INPUT data which contains the PEAR instructions, two further pieces of information have to be supplied:

DTYPE = 1 Radial structure

= 2 Love wave structure

= 3 Rayleigh wave structure

DNUMBR - the number of layers

The Love and Rayleigh wave structure are flat layer structures while the Radial structure consists of spherical shells.

For reference we give the locations in EARTH(N1,N2)

DTYPE	1	2	3

N2

Thickness (radius)	1	6	11
S velocity	2	7	12
P velocity	3	8	13
Density	4	9	14
Q	5	10	15

The location EARTH(J,5) contains QBETA for body waves; while EARTH(J,10) and EARTH(J,15) refer to QBETA AND QALPHA in the Jth layer for surface wave Q.

EXAMPLE:

```
CROSS SECTION DATA
         1         16
     IPTE                    1         1  6371.000
     IPTE                    2         1  6356.000
     IPTE                    3         1  6356.000
     IPTE                    4         1  6336.000
     IPTE                    5         1  6336.000
     IPTE                   12         1  5700.000
     IPTE                   13         1  5610.000
     IPTE                   14         1  5475.000
     IPTE                   15         1  5375.000
     IPTE                   16         1  5150.000
     CLAE          1         5         1
     SUBH          1         1
     STRT          1         6         1    5721.0
6371.0
     STRE          1         7         1
     SUBH          1         2
     STRT          1         8         1    5721.0
6871.0
     STRE          1         9         1
     SUBH          1         3
     STRT          1        10         1    5721.0
6371.0
```

```
STRE        1          11          1
IPTE                    1          2        3.300
IPTE                    2          2        3.300
IPTE                   11          2        5.370
IPTE                   12          2        5.370
IPTE                   13          2        6.380
IPTE                   14          2        6.380
IPTE                   15          2        6.400
IPTE                   16          2        6.500
CLAH        1           4
STRE        1           3          2
STRE        1           4          2
CLAH        1           5
STRE        1           5          2
STRE        1           6          2
SUBH        1           6
STRE        1           7          2
STRE        1           8          2
ADDH        1           7
STRE        1           9          2
STRE        1          10          2
IPTE                    1          3        5.709
IPTE                    2          3        5.709
IPTE                   11          3       9.2901
IPTE                   12          3       9.2901
IPTE                   13          3      11.0374
IPTE                   14          3      11.0374
IPTE                   15          3      11.0374
IPTE                   16          3      11.0374
IPTE       10                                1.73
CLAE        1           3          2
MLTR        1          10
STRE        1           3          3
STRE        1           4          3
CLAE        1           5          2
MLTR        1          10
STRE        1           5          3
STRE        1           6          3
CLAE        1           7          2
MLTR        1          10
STRE        1           7          3
STRE        1           8          6
CLAE        1           9          2
MLTR        1          10
STRE        1           9          3
STRE                   10          3
IPTE                    1          4     2.574118
IPTE                    2               2.574118
IPTE                   11          4    3.6556102
IPTE                   12          4    3.6556102
IPTE                   13          4    4.1832948
```

```
IPTE                    14      4   4.1832948
IPTE                    15      4   4.1937440
IPTE                    16      4   4.2459900
IPTR          11                        .302
IPTR          12                        .85
CLAE           3        3      3
MLTR                   11
ADDR           3       12
STRE           3        3      4
STRE           3        4      4
CLAE           3        5      3
MLTR           3       11
ADDR           3       12
STRE           3        5      4
STRE           3        6      4
CLAE           3        7      3
MLTR           3       11
ADDR           3       12
STRE           3        7      4
STRE           3        8      4
CLAE           3        9      3
MLTR           3       11
ADDR           3       12
STRE           3        9      4
STRE           3       10      4
IPTE                    1      5    100.000
IPTE                    2      5    100.000
IPTE                    3      5    100.000
IPTE                    4      5    100.000
IPTE                    5      5    100.000
IPTE                    6      5     80.000
IPTE                    7      5     80.000
IPTE                    8      5     80.000
IPTE                    9      5     80.000
IPTE                   10      5    400.000
IPTE                   11      5    800.000
IPTE                   12      5    800.000
IPTE                   13      5    800.000
IPTE                   14      5   2000.000
IPTE                   15      5   2000.000
IPTE                   16      5   2500.000
END
```

This illustrates

a) The use of H commands to generate interpolated layers in

structures as well as tests on structure through STRT

b) the use of equations of state to determine P wave velocities from S wave velocities and densities from P wave velocities

c) the straight forward setting up of unperturbed Earth structure factors.

5 THE THEORY OF THE HEDGHOG TECHNIQUE

This section describes more fully the concept of a cross-section and the use of parameters to describe Earth models and also the Hedgehog technique itself. The description is based on Keilis-Borok(1972) .

It is practically impossible to represent any of the functions alpha, beta, density, Q analytically in the whole depth interval, and in any case the functions may be discontinuous. We therefore have to divide the structure into some depth intervals (layers), and separately approximate each physical function in each layer, by the same type of function if possible. The

parameters of the structure will be the parameters of the approximating functions as well as the depth of the layer boundaries.

Parametrisation must be correct, i.e. correlated with the physical task and with the pecularities of the observational data being used. If the object is to find out whether some elements of the structure exist (low-velocity layer, or a discontinuity of velocity within some depth interval, etc,), then the assumed system of parameters has to allow structures both with these pecularities and without them. The probability of acheiving the desired pecularity by random methods must be about a half.

Clearly for correct and optimal parametrisation it is important to investigate as a start which features of the observational data are connected with the different properties of the structure.

Any measure of the discrepancy between observations and the computed properties of seismic waves depend on the parameters of the structure, because the computed properties depend on them. Our solution is a set of combinations of the unknown parameters (i.e. a volume in the space of the unknown parameters) for which the discrepancy is sufficiently small.

The simplest and the only absolutely reliable method is to divide the investigated multidimensional region by a net, to calculate the function at each node of the net and to choose that point in which the function is sufficiently small. The simplest method of construction of the net is to place the nodes along coordinate axes at equal intervals. The step of the net must have a magnitude of the order of the error that is allowed for in the determination of the boundaries of our minimum region.

If the number of parameters is large the number of nodes becomes so great that the described method is practically impossible. Thus some form of guided search method becomes attractive and the most generally useful has been found to be the 'Hedgehog' method.

A cross section is defined as the set of parameter values describing in some way an Earth structure.

The Hedgehog technique procedes as follows: a single point of the minimum region Y [Y(1),.....,Y(N)] - the Y(i)'S being the parameters of the cross section As found by some means e.g. the Monte-Carlo technique; and then the neighbouring points Y(j) + sum over (j) alpha(j) dY(j) are tried where alpha(j) = 0 or ± 1 and different combinations of the alpha(j) are tried in turn, the dy

above are the step lengths on the net assumed to be embedded in the parameter space. Points falling within this minimum region are then selected and the same procedure applied to these points until the whole region is covered. When this has been done we return to the Monte-Carlo technique (omitting of course the region just found from further investigation) and look for another minimum region, and so on.

The most reliable approach is to test all combinations of the alpha(j) i.e. to test all the neighbouring points which can be obtained from an initial point by not more than one step along each coordinate axis. This is however is computationally difficult for large N : each initial point having (3(N)-1) such neighbours. For this reason the method includes the possibility of making the limitation that only those neighbouring points for which not more than m (<N) of the alpha(j) are non zero are tested. Thus only those points which may be reached by taking on step along not more than m coordinate axes from the initial point are tested.

FIGURE 5 illustrates the Hedgehog method for a two-dimensional space. Heavy lines represent a grid for which the horizontal increment is twice that for the thin lines. The area of the minima consists of two closed parts A and B. Suppose that we found point 1 by the

Monte-Carlo technique, assume m = 1 ; then we should try the neighbouring points 2, 3, 4, 5. Point 4 belongs to our minimum region, so we then try points 6, 7, 8 around it.

To find the rest of our minima we have to return to a random search until we find the points 9, or 10 and 11 or 12. However we would think of points 1, 2 and 9, 10 as belonging to isolated regions; until we check by taking finer steps along the dotted path or introduce the finer grid. For m = N = 2 we would however try at once the points 2,3,4,5,6,7,9,13 around point 1, and then points 10,11 would be found from 9; in this case we would think of A,B as a simple area until a finer grid was introduced.

This example shows that if the increments dy(j) are too big or m is too small, we can, easily miss some points of the area of minima or introduce a fictitious division of this area into isolated parts. However decreasing dy(j) and increasing m leads to a large increase in the number of computations. Decreasing dy(j) is particularly inconvenient: it gives a lot of internal points in the area of minima which are of no interest. It is better to increase m or preferably to rotate the cordinate axes.

6 THE CONSTRUCTION OF A DATA SET

We describe here a typical example which gives an indication of what is required. An examination of the test routines in KBCAM.DATA.TEST will give the details of suitable data sets for most purposes.

COMMENT: Title of data set etc.

CLEAR: Initialising array values

CONLIM: This section sets up various limits see comments in Appendix 1 for details

PROGRAM PROCEDURE:
discussed in section 3

PARAMETERS:
Input for HGEHOG

CROSS-SECTION DATA:
PEAR language routine discussed in section 4

SYSOUT: Sets up ouput units for various parts of the package (see GROUPS in comments)

OUTPLG: Determines amount of output produced in various parts of the package (see GROUPS in comments - Appendix 1)

BODY: Input of SHwave station data and observed phases, also precision of data

P BO: Input of P wave station data and observed phases, also precision of data

ICNL: Sets the limits for ICNLIM which determines how many observation exceed the limits imposed by Conlim (see Comments in Blockdata)

† GRAV: Input of Gravity data

LOVE DATA:

Experimental phase velocities and periods or frequencies at which measured also precision of data

RAYLEIGH DATA:

Experimental phase velocities and periods or frequencies at which measured also precision of data

* QDATA: Love and Rayleigh 1/Q data and its precision

END OF DATA:

End of this input segment

STOP:

End of program

The other alternatives are listed in section 2.1 under the routine INPUT, but a sequence such as the above covers most of what is commonly useful - of course not all these groups have to be present. The details of the formats required are to be found in the source program.

* A feature of KBCAM not implemented at University of Alberta
† A feature under development